Problem 2A.1

Thickness of a falling film. Water at 20°C is flowing down a vertical wall with Re = 10. Calculate **(a)** the flow rate, in gallons per hour per foot of wall width, and **(b)** the film thickness in inches.

Answer: **(a)** 0.727 gal/hr · ft; **(b)** 0.00361 in

Solution

The Reynolds number is related to the film thickness δ and the average velocity $\langle v_z \rangle$ by Eq. 2.2-25 on page 47.

$$\mathrm{Re} = \frac{4\delta\langle v_z\rangle\rho}{\mu} \tag{2.2-25}$$

We have

$$\frac{4\delta\langle v_z\rangle\rho}{\mu} = 10.$$

Solve this for δ.

$$\delta = \frac{5}{2}\frac{\mu}{\rho\langle v_z\rangle} \tag{1}$$

Part (a)

The mass flow rate w of a falling film is given by Eq. 2.2-21 on page 46.

$$w = \rho W\delta\langle v_z\rangle \tag{2.2-21}$$

Divide both sides by W to obtain the mass flow rate per unit of wall width.

$$\frac{w}{W} = \rho\delta\langle v_z\rangle$$

Use equation (1) here.

$$\begin{aligned} &= \rho\cdot\frac{5}{2}\frac{\mu}{\rho\langle v_z\rangle}\langle v_z\rangle \\ &= \frac{5\mu}{2} \end{aligned}$$

μ, the viscosity of water, at 20°C can be found in Table 1.1-2 on page 14: $\mu = 1.0019$ mPa · s (centipoise). Use the conversion factor in Table F.3-4 on page 870 to convert this to $\mathrm{lb}_m/\mathrm{ft}\cdot\mathrm{hr}$.

$$\frac{w}{W} = \frac{5}{2}\cdot 1.0019\ \cancel{\text{centipoise}}\times\frac{2.4191\ \frac{\mathrm{lb}_m}{\mathrm{ft\cdot hr}}}{1\ \cancel{\text{centipoise}}} \approx 6.0592\ \frac{\mathrm{lb}_m}{\mathrm{ft\cdot hr}}$$

We have to divide this result by the density of water to get units of volume in the numerator. Before we do, find the density in units of lb_m/gal, starting with the known value 1000 kg/m^3 and using the conversion factors on page 868.

$$1000\ \frac{\cancel{\mathrm{kg}}}{\cancel{\mathrm{m}^3}}\times\frac{2.2046\ \mathrm{lb}_m}{1\ \cancel{\mathrm{kg}}}\times\left(\frac{1\ \cancel{\mathrm{m}}}{3.28\ \cancel{\mathrm{ft}}}\right)^3\times\frac{0.13368\ \cancel{\mathrm{ft}^3}}{1\ \mathrm{gal}} \approx 8.345\ \frac{\mathrm{lb}_m}{\mathrm{gal}}$$

Therefore, we have for the flow rate

$$\frac{w}{\rho W} \approx 6.0592 \frac{\cancel{\text{lb}_m}}{\text{ft} \cdot \text{hr}} \times \frac{1 \text{ gal}}{8.345 \cancel{\text{lb}_m}} \approx 0.726 \frac{\text{gal}}{\text{hr} \cdot \text{ft}}.$$

Part (b)

The film thickness is given by equation (1) above. The average velocity $\langle v_z \rangle$ of the film is given by Eq. 2.2-20 on page 45.

$$\langle v_z \rangle = \frac{\rho g \delta^2 \cos \beta}{3\mu} \tag{2.2-20}$$

β represents the angle from the vertical that the film is flowing down. Since the wall is vertical, $\beta = 0$ and $\cos \beta = 1$. Substitute this expression for $\langle v_z \rangle$ into equation (1).

$$\begin{aligned}
\delta &= \frac{5\mu}{2\rho} \cdot \frac{3\mu}{\rho g \delta^2} \\
\delta^3 &= \frac{15\mu^2}{2\rho^2 g} \\
\delta &= \sqrt[3]{\frac{15\mu^2}{2\rho^2 g}} \\
&\approx \sqrt[3]{\frac{15(1.0019 \times 10^{-3} \text{ Pa})^2}{2(1000 \text{ kg/m}^3)^2 (9.81 \text{ m/s}^2)}} \\
&\approx 9.16 \times 10^{-5} \cancel{\text{m}} \times \frac{3.28 \cancel{\text{ft}}}{1 \cancel{\text{m}}} \times \frac{12 \text{ in}}{1 \cancel{\text{ft}}}
\end{aligned}$$

Therefore,

$$\delta \approx 0.00360 \text{ in.}$$