# Problem 3B.7

**Momentum fluxes for creeping flow into a slot** (Fig. 3B.7). An incompressible Newtonian liquid is flowing very slowly into a thin slot of thickness $2B$ (in the $y$ direction) and width $W$ (in the $z$ direction). The mass rate of flow in the slot is $w$. From the results of Problem 2B.3 it can be shown that the velocity distribution within the slot is

$$v_x = \frac{3w}{4BW\rho}\left[1 - \left(\frac{y}{B}\right)^2\right] \quad v_y = 0 \quad v_z = 0 \tag{3B.7-1}$$

at locations not too near the inlet. In the region outside the slot the components of the velocity for *creeping flow* are

$$v_x = -\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2} \tag{3B.7-2}$$

$$v_y = -\frac{2w}{\pi W\rho}\frac{x^2 y}{(x^2+y^2)^2} \tag{3B.7-3}$$

$$v_z = 0 \tag{3B.7-4}$$

Equations 3B.7-1 to 4 are only approximate in the region near the slot entry for both $x \geq 0$ and $x \leq 0$.

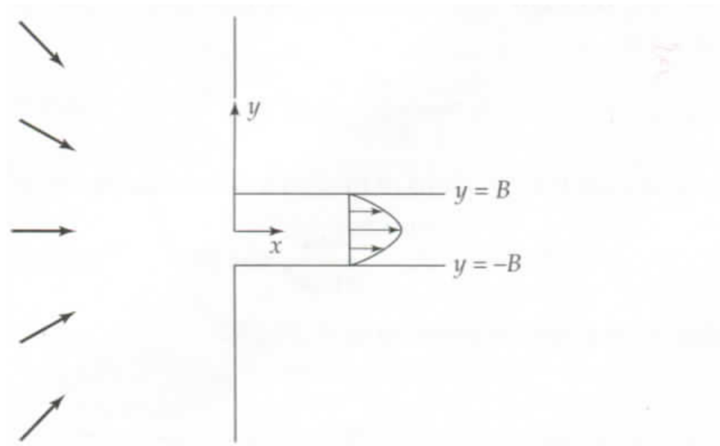


**Fig. 3B.7.** Flow of a liquid into a slot from a semi-infinite region $x < 0$.

**(a)** Find the components of the convective momentum flux $\rho\mathbf{vv}$ inside and outside the slot.

**(b)** Evaluate the $xx$-component of $\rho\mathbf{vv}$ at $x = -a$, $y = 0$.

**(c)** Evaluate the $xy$-component of $\rho\mathbf{vv}$ at $x = -a$, $y = +a$.

**(d)** Does the total flow of kinetic energy through the plane $x = -a$ equal the total flow of kinetic energy through the slot?

**(e)** Verify that the velocity distributions given in Eqs. 3B.7-1 to 4 satisfy the relation $(\nabla \cdot \mathbf{v}) = 0$.

**(f)** Find the normal stress $\tau_{xx}$ at the plane $y = 0$ and also on the solid surface at $x = 0$.

**(g)** Find the shear stress $\tau_{yx}$ on the solid surface at $x = 0$. Is this result surprising? Does sketching the velocity profile $v_y$ vs. $x$ at some plane $y = a$ assist in understanding the result?

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**Solution**

The convective momentum flux, a second-order tensor, is

$$\begin{aligned}\rho\mathbf{vv} &= \rho\left(\sum_{i=1}^{3} v_i\boldsymbol{\delta}_i\right)\left(\sum_{j=1}^{3} v_j\boldsymbol{\delta}_j\right)\\ &= \rho(v_1\boldsymbol{\delta}_1 + v_2\boldsymbol{\delta}_2 + v_3\boldsymbol{\delta}_3)(v_1\boldsymbol{\delta}_1 + v_2\boldsymbol{\delta}_2 + v_3\boldsymbol{\delta}_3)\\ &= \rho(v_1^2\boldsymbol{\delta}_1\boldsymbol{\delta}_1 + v_1v_2\boldsymbol{\delta}_1\boldsymbol{\delta}_2 + v_1v_3\boldsymbol{\delta}_1\boldsymbol{\delta}_3\\ &\quad + v_1v_2\boldsymbol{\delta}_2\boldsymbol{\delta}_1 + v_2^2\boldsymbol{\delta}_2\boldsymbol{\delta}_2 + v_2v_3\boldsymbol{\delta}_2\boldsymbol{\delta}_3\\ &\qquad + v_1v_3\boldsymbol{\delta}_3\boldsymbol{\delta}_1 + v_2v_3\boldsymbol{\delta}_3\boldsymbol{\delta}_2 + v_3^2\boldsymbol{\delta}_3\boldsymbol{\delta}_3).\end{aligned}$$

## Part (a)

The components of velocity inside the slot are

$$\begin{aligned}v_x &= \frac{3w}{4BW\rho}\left[1 - \left(\frac{y}{B}\right)^2\right]\\ v_y &= 0\\ v_z &= 0,\end{aligned}$$

so the convective momentum flux for $x > 0$ is

$$\begin{aligned}\rho\mathbf{vv} &= \rho v_1^2\boldsymbol{\delta}_1\boldsymbol{\delta}_1\\ &= \rho\frac{9w^2}{16B^2W^2\rho^2}\left[1 - \left(\frac{y}{B}\right)^2\right]^2\boldsymbol{\delta}_x\boldsymbol{\delta}_x\\ &= \frac{9w^2}{16B^2W^2\rho}\left(1 - \frac{y^2}{B^2}\right)^2\boldsymbol{\delta}_x\boldsymbol{\delta}_x.\end{aligned}$$

On the other hand, the components of velocity outside the slot are

$$\begin{aligned}v_x &= -\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\\ v_y &= -\frac{2w}{\pi W\rho}\frac{x^2y}{(x^2+y^2)^2}\\ v_z &= 0,\end{aligned}$$

so the convective momentum flux for $x < 0$ is

$$\begin{aligned}\rho\mathbf{vv} &= \rho(v_1^2\boldsymbol{\delta}_1\boldsymbol{\delta}_1 + v_1v_2\boldsymbol{\delta}_1\boldsymbol{\delta}_2\\ &\quad + v_1v_2\boldsymbol{\delta}_2\boldsymbol{\delta}_1 + v_2^2\boldsymbol{\delta}_2\boldsymbol{\delta}_2).\end{aligned}$$

Substitute the components and expand the right side.

$$\begin{aligned}
\rho\mathbf{vv} &= \rho\left[-\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\right]^2\boldsymbol{\delta}_x\boldsymbol{\delta}_x + \rho\left[-\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\right]\left[-\frac{2w}{\pi W\rho}\frac{x^2y}{(x^2+y^2)^2}\right]\boldsymbol{\delta}_x\boldsymbol{\delta}_y \\
&\quad + \rho\left[-\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\right]\left[-\frac{2w}{\pi W\rho}\frac{x^2y}{(x^2+y^2)^2}\right]\boldsymbol{\delta}_y\boldsymbol{\delta}_x + \rho\left[-\frac{2w}{\pi W\rho}\frac{x^2y}{(x^2+y^2)^2}\right]^2\boldsymbol{\delta}_y\boldsymbol{\delta}_y \\
&= \frac{4w^2}{\pi^2W^2\rho}\frac{x^6}{(x^2+y^2)^4}\boldsymbol{\delta}_x\boldsymbol{\delta}_x + \frac{4w^2}{\pi^2W^2\rho}\frac{x^5y}{(x^2+y^2)^4}\boldsymbol{\delta}_x\boldsymbol{\delta}_y \\
&\quad + \frac{4w^2}{\pi^2W^2\rho}\frac{x^5y}{(x^2+y^2)^4}\boldsymbol{\delta}_y\boldsymbol{\delta}_x + \frac{4w^2}{\pi^2W^2\rho}\frac{x^4y^2}{(x^2+y^2)^4}\boldsymbol{\delta}_y\boldsymbol{\delta}_y
\end{aligned}$$

Therefore,

$$\rho\mathbf{vv} = \begin{cases} \frac{4w^2}{\pi^2W^2\rho}\frac{x^6}{(x^2+y^2)^4}\boldsymbol{\delta}_x\boldsymbol{\delta}_x + \frac{4w^2}{\pi^2W^2\rho}\frac{x^5y}{(x^2+y^2)^4}\boldsymbol{\delta}_x\boldsymbol{\delta}_y & \\ \quad + \frac{4w^2}{\pi^2W^2\rho}\frac{x^5y}{(x^2+y^2)^4}\boldsymbol{\delta}_y\boldsymbol{\delta}_x + \frac{4w^2}{\pi^2W^2\rho}\frac{x^4y^2}{(x^2+y^2)^4}\boldsymbol{\delta}_y\boldsymbol{\delta}_y & \text{if } x<0 \\ \frac{9w^2}{16B^2W^2\rho}\left(1-\frac{y^2}{B^2}\right)^2\boldsymbol{\delta}_x\boldsymbol{\delta}_x & \text{if } x>0 \end{cases}.$$

Note that physically each term represents the convective flux of (2nd delta subscript)-momentum across a plane that is perpendicular to the (1st delta subscript)-direction. Observe that some of them are always positive (even for flows different than the one in this problem). The $\boldsymbol{\delta}_x\boldsymbol{\delta}_x$ fluxes are because the liquid always flows from left to right in the positive $x$-direction. The $\boldsymbol{\delta}_y\boldsymbol{\delta}_y$ flux is as well because even though the liquid travels up in the positive $y$-direction only for $y<0$, the liquid travels down in the negative $y$-direction for $y>0$. The $\boldsymbol{\delta}_x\boldsymbol{\delta}_y$ flux is positive when the liquid travels up for $y<0$ and goes in the positive $x$-direction but is negative when the liquid travels down for $y>0$ and goes in the positive $x$-direction. The $\boldsymbol{\delta}_y\boldsymbol{\delta}_x$ flux is positive when the liquid travels to the right and goes in the positive $y$-direction ($y<0$) but is negative when the liquid travels to the right and goes in the negative $y$-direction ($y>0$).

## Part (b)

$x=-a$ is outside the slot, so the relevant $xx$-component of $\rho\mathbf{vv}$ is

$$\frac{4w^2}{\pi^2W^2\rho}\frac{x^6}{(x^2+y^2)^4}.$$

Plug in $x=-a$ and $y=0$ and simplify the result.

$$\frac{4w^2}{\pi^2W^2\rho}\frac{a^6}{a^8}$$

$$\frac{4w^2}{\pi^2a^2W^2\rho}$$

**Part (c)**

$x = -a$ is outside the slot, so the $xy$-component of $\rho\mathbf{vv}$ is

$$\frac{4w^2}{\pi^2 W^2 \rho} \frac{x^5 y}{(x^2+y^2)^4}.$$

Plug in $x = -a$ and $y = a$ and simplify the result.

$$\frac{4w^2}{\pi^2 W^2 \rho} \frac{(-a)^5(a)}{(2a^2)^4}$$

$$\frac{4w^2}{\pi^2 W^2 \rho} \frac{-a^6}{16a^8}$$

$$-\frac{w^2}{4\pi^2 a^2 W^2 \rho}$$

**Part (d)**

For a liquid in motion, the kinetic energy per unit volume is

$$\frac{1}{2}\rho v^2.$$

Multiply this by the volumetric flow rate (volume per unit time) to get the kinetic energy flow.

$$\frac{1}{2}\rho v^2 \frac{dV}{dt}$$

$dV/dt$ is the dot product of velocity and area.

$$\frac{1}{2}\rho v^2 (\mathbf{v} \cdot \mathbf{A})$$

Actually, because the velocity varies with $x$ and $y$, it's necessary to integrate over the area that the liquid flows through.

$$\int \frac{1}{2}\rho v^2 (\mathbf{v} \cdot d\mathbf{A})$$

Begin by calculating the kinetic energy flow through the $x = -a$ plane. The unit vector perpendicular to this plane is $\boldsymbol{\delta}_x$, which means the dot product yields the $x$-component of velocity.

$$\int \frac{1}{2}\rho v^2 v_x \, dA$$

Integrate over the entire $x = -a$ plane.

$$\int_{-\infty}^{\infty} \frac{1}{2}\rho v^2 v_x (W\, dy)$$

Bring the constants in front and rewrite $v^2$.

$$\frac{\rho W}{2} \int_{-\infty}^{\infty} (v_x^2 + v_y^2 + v_z^2) v_x \, dy$$

Substitute the components of velocity outside the slot and evaluate the integral.

$$\frac{\rho W}{2}\int_{-\infty}^{\infty}\left[\frac{4w^2}{\pi^2W^2\rho^2}\frac{x^6}{(x^2+y^2)^4}+\frac{4w^2}{\pi^2W^2\rho^2}\frac{x^4y^2}{(x^2+y^2)^4}+0\right]\left[-\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\right]dy$$

$$\frac{\rho W}{2}\int_{-\infty}^{\infty}\left[\frac{x^6}{(x^2+y^2)^4}+\frac{x^4y^2}{(x^2+y^2)^4}\right]\left[-\frac{8w^3}{\pi^3W^3\rho^3}\frac{x^3}{(x^2+y^2)^2}\right]dy$$

$$-\frac{4w^3}{\pi^3W^2\rho^2}\int_{-\infty}^{\infty}\left[\frac{x^9}{(x^2+y^2)^6}+\frac{x^7y^2}{(x^2+y^2)^6}\right]dy$$

Substitute $x=-a$.

$$-\frac{4w^3}{\pi^3W^2\rho^2}\int_{-\infty}^{\infty}\left[\frac{-a^9}{(a^2+y^2)^6}+\frac{-a^7y^2}{(a^2+y^2)^6}\right]dy$$

$$\frac{4w^3}{\pi^3W^2\rho^2}\left[a^9\int_{-\infty}^{\infty}\frac{dy}{(a^2+y^2)^6}+a^7\int_{-\infty}^{\infty}\frac{y^2}{(a^2+y^2)^6}\,dy\right]$$

$$\frac{4w^3}{\pi^3W^2\rho^2}\left(a^9\cdot\frac{63\pi}{256a^{11}}+a^7\cdot\frac{7\pi}{256a^9}\right)$$

$$\frac{4w^3}{\pi^3W^2\rho^2}\left(\frac{35\pi}{128a^2}\right)$$

$$\boxed{\frac{35w^3}{32\pi^2a^2W^2\rho^2}}$$

Again, this is the flow of kinetic energy through the $x=-a$ plane, which is outside the slot. Now the kinetic energy flow through the inside of the slot will be calculated.

$$\int\frac{1}{2}\rho v^2(\mathbf{v}\cdot d\mathbf{A})$$

For any plane $x=b>0$ in the slot, the unit vector perpendicular to it is $\boldsymbol{\delta}_x$; this makes the dot product yield $v_x$.

$$\int\frac{1}{2}\rho v^2v_x\,dA$$

Integrate over the entire $x=b$ plane.

$$\int_{-B}^{B}\frac{1}{2}\rho v^2v_x(W\,dy)$$

Bring the constants in front and rewrite $v^2$.

$$\frac{\rho W}{2}\int_{-B}^{B}(v_x^2+v_y^2+v_z^2)v_x\,dy$$

Substitute the components of velocity inside the slot.

$$\frac{\rho W}{2}\int_{-B}^{B}\left[\frac{9w^2}{16B^2W^2\rho^2}\left(1-\frac{y^2}{B^2}\right)^2+0+0\right]\left[\frac{3w}{4BW\rho}\left(1-\frac{y^2}{B^2}\right)\right]dy$$

$$\frac{\rho W}{2}\int_{-B}^{B}\left[\frac{27w^3}{64B^3W^3\rho^3}\left(1-\frac{y^2}{B^2}\right)^3\right]dy$$

Bring the constants in front and evaluate the integral.

$$\frac{27w^3}{128B^3W^2\rho^2}\int_{-B}^{B}\left(1-\frac{y^2}{B^2}\right)^3 dy$$

$$\frac{27w^3}{128B^3W^2\rho^2}\int_{-B}^{B}\left(1-\frac{y^2}{B^2}\right)^3 dy$$

$$\frac{27w^3}{128B^3W^2\rho^2}\left(\frac{32B}{35}\right)$$

$$\boxed{\frac{27w^3}{140B^2W^2\rho^2}}$$

Again, this is the flow of kinetic energy inside the slot. Therefore, the flow of kinetic energy through the plane $x = -a$ is not equal to the flow of kinetic energy through the slot.

## Part (e)

Check that $\nabla \cdot \mathbf{v} = 0$ is satisfied both inside and outside the slot. The velocity components are as follows.

Inside The Slot

$$v_x = \frac{3w}{4BW\rho}\left[1-\left(\frac{y}{B}\right)^2\right]$$

$$v_y = 0$$

$$v_z = 0$$

Outside the slot

$$v_x = -\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}$$

$$v_y = -\frac{2w}{\pi W\rho}\frac{x^2 y}{(x^2+y^2)^2}$$

$$v_z = 0$$

Inside the slot, we have

$$\begin{aligned}
\nabla \cdot \mathbf{v} &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right\rangle \cdot \langle v_x, v_y, v_z\rangle \\
&= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
&= \frac{\partial}{\partial x}\frac{3w}{4BW\rho}\left[1-\left(\frac{y}{B}\right)^2\right] + \frac{\partial}{\partial y}(0) + \frac{\partial}{\partial z}(0) \\
&= \frac{3w}{4BW\rho}\frac{\partial}{\partial x}\left[1-\left(\frac{y}{B}\right)^2\right] + 0 + 0 \\
&= \frac{3w}{4BW\rho}(0) \\
&= 0.
\end{aligned}$$

Outside the slot, we have

$$\begin{aligned}
\nabla \cdot \mathbf{v} &= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle \cdot \langle v_x, v_y, v_z \rangle \\
&= \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} \\
&= \frac{\partial}{\partial x}\left[-\frac{2w}{\pi W \rho}\frac{x^3}{(x^2+y^2)^2}\right] + \frac{\partial}{\partial y}\left[-\frac{2w}{\pi W \rho}\frac{x^2 y}{(x^2+y^2)^2}\right] + \frac{\partial}{\partial z}(0) \\
&= -\frac{2w}{\pi W \rho}\frac{\partial}{\partial x}\left[\frac{x^3}{(x^2+y^2)^2}\right] - \frac{2w}{\pi W \rho}\frac{\partial}{\partial y}\left[\frac{x^2 y}{(x^2+y^2)^2}\right] + 0 \\
&= -\frac{2w}{\pi W \rho}\left[\frac{3x^2(x^2+y^2)^2 - 2(x^2+y^2)\cdot 2x(x^3)}{(x^2+y^2)^4}\right] - \frac{2w}{\pi W \rho}\left[\frac{x^2(x^2+y^2)^2 - 2(x^2+y^2)\cdot 2y(x^2 y)}{(x^2+y^2)^4}\right] \\
&= -\frac{2w}{\pi W \rho}\left[\frac{3x^2(x^2+y^2) - 4x(x^3)}{(x^2+y^2)^3}\right] - \frac{2w}{\pi W \rho}\left[\frac{x^2(x^2+y^2) - 4y(x^2 y)}{(x^2+y^2)^3}\right] \\
&= -\frac{2w}{\pi W \rho}\left[\frac{3x^2(x^2+y^2) - 4x(x^3) + x^2(x^2+y^2) - 4y(x^2 y)}{(x^2+y^2)^3}\right] \\
&= -\frac{2w}{\pi W \rho}\left[\frac{\cancel{3x^4} + \cancel{3x^2y^2} - \cancel{4x^4} + \cancel{x^4} + \cancel{x^2y^2} - \cancel{4x^2y^2}}{(x^2+y^2)^3}\right] \\
&= 0.
\end{aligned}$$

## Part (f)

The normal stress $\tau_{xx}$ is given on page 843.

$$\tau_{xx} = -\mu\left[2\frac{\partial v_x}{\partial x}\right] + \left(\frac{2}{3}\mu - \kappa\right)(\nabla \cdot \mathbf{v})$$

It was verified in part (e) that $\nabla \cdot \mathbf{v} = 0$.

$$\tau_{xx} = -2\mu\frac{\partial v_x}{\partial x}$$

Inside the slot $v_x$ is only a function of $y$, meaning $\tau_{xx}$ is zero for $x > 0$. Outside the slot, however, the normal stress is

$$\begin{aligned}
\tau_{xx} &= -2\mu\frac{\partial}{\partial x}\left[-\frac{2w}{\pi W \rho}\frac{x^3}{(x^2+y^2)^2}\right] \\
&= \frac{4\mu w}{\pi W \rho}\frac{\partial}{\partial x}\left[\frac{x^3}{(x^2+y^2)^2}\right] \\
&= \frac{4\mu w}{\pi W \rho}\left[\frac{3x^2(x^2+y^2)^2 - 2(x^2+y^2)\cdot 2x(x^3)}{(x^2+y^2)^4}\right] \\
&= \frac{4\mu w}{\pi W \rho}\left[\frac{3x^2(x^2+y^2) - 4x(x^3)}{(x^2+y^2)^3}\right] \\
&= \frac{4\mu w}{\pi W \rho}\frac{3x^2y^2 - x^4}{(x^2+y^2)^3}.
\end{aligned}$$

Therefore,

$$\tau_{xx} = \begin{cases} \dfrac{4\mu w}{\pi W\rho}\dfrac{3x^2y^2 - x^4}{(x^2+y^2)^3} & \text{if } x<0 \\ 0 & \text{if } x>0 \end{cases}.$$

It follows that the normal stress on the $x=0$ solid surface is

$$\lim_{x\to 0}\tau_{xx} = 0,$$

and the normal stress on the $y=0$ plane is

$$\lim_{y\to 0}\tau_{xx} = \begin{cases} -\dfrac{4\mu w}{\pi W\rho x^2} & \text{if } x<0 \\ 0 & \text{if } x>0 \end{cases}.$$

**Part (g)**

The shear stress $\tau_{yx}$ is also given on page 843.

$$\tau_{yx} = -\mu\left[\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}\right]$$

Inside the slot, we have

$$\begin{aligned}\tau_{yx} &= -\mu\left\{0 + \frac{\partial}{\partial y}\left[\frac{3w}{4BW\rho}\left(1 - \frac{y^2}{B^2}\right)\right]\right\} \\ &= -\frac{3\mu w}{4BW\rho}\left(-\frac{2y}{B^2}\right) \\ &= \frac{3\mu w}{2B^3W\rho}y.\end{aligned}$$

Outside the slot, we have

$$\begin{aligned}\tau_{yx} &= -\mu\left\{\frac{\partial}{\partial x}\left[-\frac{2w}{\pi W\rho}\frac{x^2y}{(x^2+y^2)^2}\right] + \frac{\partial}{\partial y}\left[-\frac{2w}{\pi W\rho}\frac{x^3}{(x^2+y^2)^2}\right]\right\} \\ &= -\mu\left\{-\frac{2w}{\pi W\rho}\frac{\partial}{\partial x}\left[\frac{x^2y}{(x^2+y^2)^2}\right] - \frac{2w}{\pi W\rho}\frac{\partial}{\partial y}\left[\frac{x^3}{(x^2+y^2)^2}\right]\right\} \\ &= \frac{2\mu w}{\pi W\rho}\left\{\frac{\partial}{\partial x}\left[\frac{x^2y}{(x^2+y^2)^2}\right] + \frac{\partial}{\partial y}\left[\frac{x^3}{(x^2+y^2)^2}\right]\right\} \\ &= \frac{2\mu w}{\pi W\rho}\left\{\left[\frac{2xy(x^2+y^2)^2 - 2(x^2+y^2)\cdot 2x(x^2y)}{(x^2+y^2)^4}\right] + \left[\frac{(0)(x^2+y^2)^2 - 2(x^2+y^2)\cdot 2y(x^3)}{(x^2+y^2)^4}\right]\right\} \\ &= \frac{2\mu w}{\pi W\rho}\left\{\left[\frac{2xy(x^2+y^2) - 4x(x^2y)}{(x^2+y^2)^3}\right] + \left[\frac{-4y(x^3)}{(x^2+y^2)^3}\right]\right\} \\ &= \frac{2\mu w}{\pi W\rho}\left[\frac{2xy^3 - 6x^3y}{(x^2+y^2)^3}\right] \\ &= \frac{2\mu w}{\pi W\rho}\left[\frac{2xy(y^2-3x^2)}{(x^2+y^2)^3}\right].\end{aligned}$$

Therefore,

$$\tau_{yx} = \begin{cases} \frac{2\mu w}{\pi W \rho}\left[\frac{2xy(y^2 - 3x^2)}{(x^2+y^2)^3}\right] & \text{if } x < 0 \\ \frac{3\mu w}{2B^3 W \rho} y & \text{if } x > 0 \end{cases}.$$

It follows that the shear stress on the $x = 0$ solid surface (from the left) is

$$\lim_{x \to 0^-} \tau_{yx} = 0.$$

In light of the components of velocity outside the slot,

$$v_x = -\frac{2w}{\pi W \rho}\frac{x^3}{(x^2+y^2)^2}$$
$$v_y = -\frac{2w}{\pi W \rho}\frac{x^2 y}{(x^2+y^2)^2}$$
$$v_z = 0,$$

this result for the shear stress is not surprising because $v_y$ is even in $x$ (meaning that $\partial v_y/\partial x$ is zero at $x = 0$) and $v_x$ has $x^3$ in the numerator (meaning you get zero when $\partial v_x/\partial y$ is evaluated at $x = 0$). Below is a plot of $v_y$ versus $x$ for $w = W = \rho = 1$ on the $y = 1$ plane to illustrate this first point.