Problem 4A.1

Time for attainment of steady state in tube flow.

(a) A heavy oil, with a kinematic viscosity of 3.45×10^{-4} m^2/s, is at rest in a long vertical tube with a radius of 0.7 cm. The fluid is suddenly allowed to flow from the bottom of the tube by virtue of gravity. After what time will the velocity at the tube center be within 10% of its final value?

(b) What is the result if water at 68°F is used?

Note: The result shown in Fig. 4D.2 should be used.

Answers: **(a)** 6.4×10^{-2} s; **(b)** 22 s

Solution

The problem of finding the time-dependent velocity distribution in a circular tube is solved in Problem 4D.2. In terms of dimensionless variables, the velocity is

$$\phi(\xi, \tau) = (1 - \xi^2) - 8 \sum_{n=1}^{\infty} \frac{J_0(\alpha_n \xi)}{\alpha_n^3 J_1(\alpha_n)} \exp(-\alpha_n^2 \tau),$$

and a plot of it for various values of τ is shown in Fig. 4D.2.

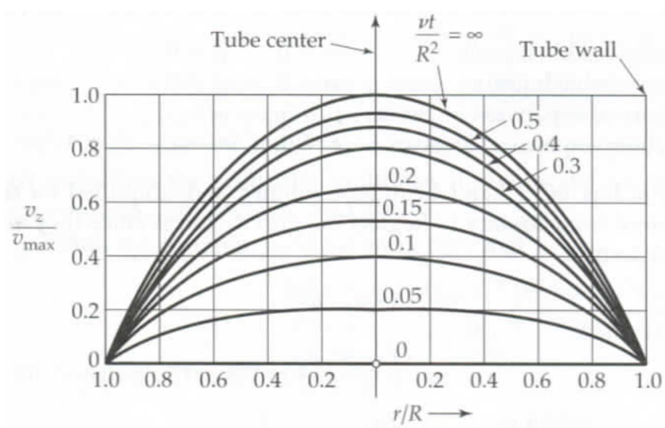

Fig. 4D.2. Velocity distribution for the unsteady flow resulting from a suddenly impressed pressure gradient in a circular tube [P. Szymanski, *J. Math. Pures Appl.*, Series 9, **11**, 67–107 (1932)].

Part (a)

According to Fig. 4D.2, the velocity at the tube center reaches 90% of its final value at the steady state when τ is somewhere between 0.4 and 0.5, say 0.45. Therefore,

$$\tau = \frac{\nu t}{R^2} = 0.45 \quad \Rightarrow \quad t = \frac{0.45 R^2}{\nu} = \frac{0.45 \left(0.7\ \cancel{\text{cm}} \times \frac{1\ \text{m}}{100\ \cancel{\text{cm}}}\right)^2}{3.45 \times 10^{-4}\ \frac{\text{m}^2}{\text{s}}} \approx 0.0639\ \text{s}$$

for the oil.

Part (b)

Water at 68°F is 20°C by using the conversion formula,

$$F = \frac{9}{5}C + 32.$$

From Table 1.1-2 on page 14, the kinematic viscosity of water at this temperature is $\nu = 0.010037 \text{ cm}^2/\text{s}$. Therefore, at 68°F the time at which the water velocity reaches 90% of its value at the steady state is

$$\tau = \frac{\nu t}{R^2} = 0.45 \quad \Rightarrow \quad t = \frac{0.45R^2}{\nu} = \frac{0.45(0.7 \text{ cm})^2}{0.010037 \frac{\text{cm}^2}{\text{s}}} \approx 22.0 \text{ s}.$$