Problem 4A.7

Entrance flow in conduits.

(a) Estimate the entrance length for laminar flow in a circular tube. Assume that the boundary-layer thickness δ is given adequately by Eq. 4.4-17, with v_∞ of the flat-plate problem corresponding to $v_{\max}$ in the tube-flow problem. Assume further that the entrance length L_e can be taken to be the value of x at which $\delta = R$. Compare your result with the expression for L_e cited in §2.3—namely, $L_e = 0.035D\,\text{Re}$.

(b) Rewrite the transition Reynolds number $xv_\infty/\nu \approx 3.5 \times 10^5$ (for the flat plate) by inserting δ from Eq. 4.4-17 in place of x as the characteristic length. Compare the quantity $\delta v_\infty/\nu$ thus obtained with the corresponding minimum transition Reynolds number for the flow through long smooth tubes.

(c) Use the method of (a) to estimate the entrance length in the flat duct shown in Fig. 4C.1. Compare the result with that given in Problem 4C.1(d).

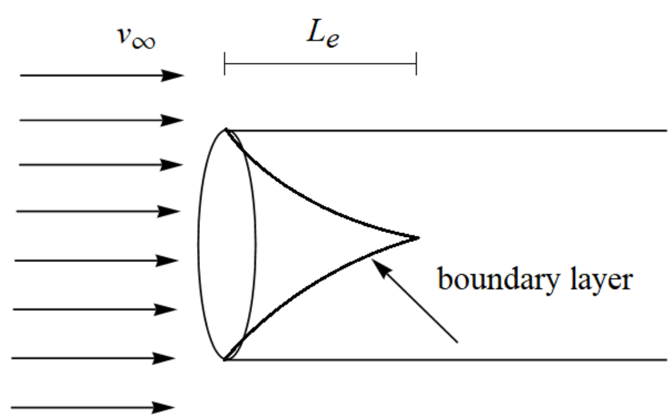
Solution

Part (a)

The boundary layer thickness due to tangential laminar flow over a flat plate is given by Eq. 4.4-17 on page 137.

$$\delta(x) = \sqrt{\frac{280}{13}\frac{\nu x}{v_\infty}} \tag{4.4-17}$$

Here it will be used to approximately determine the entrance length L_e in a cylindrical tube.



At $x = L_e$ the boundary layer thickness is the radius of the tube: $\delta(L_e) = R$.

$$R = \sqrt{\frac{280}{13}\frac{\nu L_e}{v_\infty}}$$

Square both sides and solve for L_e.

$$R^2 = \frac{280}{13}\frac{\nu L_e}{v_\infty}$$

$$L_e = \frac{13}{280} R \frac{R v_\infty}{\nu}$$

v_∞ of the flat-plate problem is assumed to correspond to $v_{\max}$ of the tube-flow problem.

$$L_e = \frac{13}{280} R \frac{R v_{\max}}{\nu}$$

According to page 51 (Eq. 2.3-20), the maximum velocity of a flow in a tube is double the average velocity: $v_{\max} = 2\langle v \rangle$.

$$L_e = \frac{13}{280} R \frac{2R\langle v \rangle}{\nu}$$

Use the diameter D for $2R$.

$$L_e = \frac{13}{280} R \frac{D\langle v \rangle}{\nu}$$

According to the top of page 52, the Reynolds number for flow in a tube is $\mathrm{Re} = D\langle v \rangle \rho/\mu$. Therefore, the entrance length in a cylindrical tube is

$$\begin{aligned} L_e &= \frac{13}{280} R \,\mathrm{Re} \\ &= \frac{13}{560} 2R \,\mathrm{Re} \\ &= \frac{13}{560} D \,\mathrm{Re} \\ &\approx 0.023 D \,\mathrm{Re}, \end{aligned}$$

which has a percent difference of

$$\frac{\frac{13}{560} D \,\mathrm{Re} - 0.035 D \,\mathrm{Re}}{0.035 D \,\mathrm{Re}} \times 100\% \approx -34\%$$

compared to the value given in §2.3 on page 52, $L_e = 0.035 D \,\mathrm{Re}$.

Part (b)

The aim here is to estimate the Reynolds number for which laminar flow in a tube becomes turbulent. The transition Reynolds number for a flat plate is $x v_\infty/\nu$. Use δ in place of x.

$$\begin{aligned} \frac{\delta v_\infty}{\nu} &= \sqrt{\frac{280}{13} \frac{\nu x}{v_\infty}} \cdot \frac{v_\infty}{\nu} \\ &= \sqrt{\frac{280}{13} \frac{x v_\infty}{\nu}} \\ &= \sqrt{\frac{280}{13} (3.5 \times 10^5)} \\ &\approx 2700 \end{aligned}$$

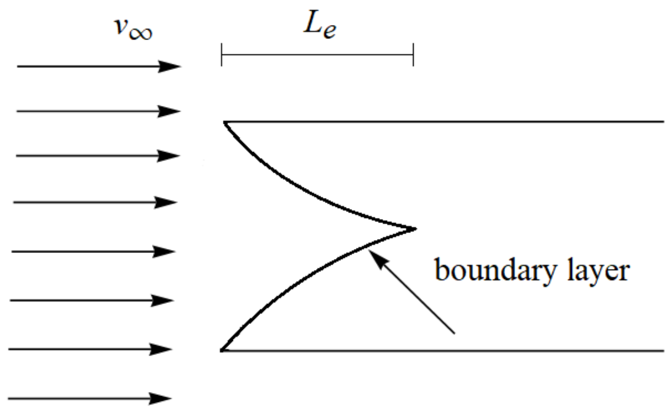
This has a percent difference of

$$\frac{2700 - 2100}{2100} \times 100\% \approx +31\%$$

compared to the value of 2100 given in §2.3 on page 52.

Part (c)

The flow considered in Problem 4C.1 occurs at the entrance of a narrow slit, which is formed by two parallel plates.



The boundary layer thickness due to tangential laminar flow over a flat plate is given by Eq. 4.4-17 on page 137.

$$\delta(x) = \sqrt{\frac{280}{13}\frac{\nu x}{v_\infty}} \tag{4.4-17}$$

Here it will be used to approximately determine the entrance length L_e in a slit. At $x = L_e$ the boundary layer thickness is half the length of the slit: $\delta(L_e) = B$.

$$B = \sqrt{\frac{280}{13}\frac{\nu L_e}{v_\infty}}$$

Square both sides and solve for L_e.

$$B^2 = \frac{280}{13}\frac{\nu L_e}{v_\infty}$$

$$L_e = \frac{13}{280}\frac{v_\infty B^2}{\nu}$$

v_∞ of the flat-plate problem is assumed to correspond to $v_{\max}$ of the slit-flow problem.

$$L_e = \frac{13}{280}\frac{v_{\max} B^2}{\nu}$$

According to Problem 2B.3, the maximum velocity of a flow in a slit is 1.5 times the average velocity: $v_{\max} = \frac{3}{2}\langle v_x\rangle$. Therefore, the entrance length in a narrow slit is

$$L_e = \frac{13}{280}\frac{\frac{3}{2}\langle v_x\rangle B^2}{\nu} = \frac{39}{560}\frac{\langle v_x\rangle B^2}{\nu} \approx 0.0696\frac{\langle v_x\rangle B^2}{\nu},$$

which has a percent difference of

$$\frac{0.0696\frac{\langle v_x\rangle B^2}{\nu} - 0.104\frac{\langle v_x\rangle B^2}{\nu}}{0.104\frac{\langle v_x\rangle B^2}{\nu}} \times 100\% \approx -33.0\%$$

compared to the value given in Problem 4C.1 on page 146, $L_e = 0.104\langle v_x\rangle B^2/\nu$.