# Problem 1.1

Solve the separable equations:

$$(a) \quad y' = e^{x+y};$$
$$(b) \quad y' = xy + x + y + 1$$

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**Solution**

## Part (a)

This differential equation can be solved by separation of variables because the right-hand side can be factored.

$$\begin{aligned} y' &= e^{x+y} \\ \frac{dy}{dx} &= e^x e^y \\ \frac{dy}{e^y} &= e^x \, dx \\ \int e^{-y} \, dy &= \int e^x \, dx \\ -e^{-y} &= e^x + C \\ e^{-y} &= -e^x - C \\ -y &= \ln(-e^x - C) \end{aligned}$$

Therefore,

$$y(x) = \ln\left(-\frac{1}{e^x + C}\right).$$

## Part (b)

This differential equation can be solved by separation of variables as well because the right-hand side can be factored.

$$\begin{aligned} y' &= xy + x + y + 1 \\ \frac{dy}{dx} &= y(x+1) + x + 1 \\ \frac{dy}{dx} &= (y+1)(x+1) \\ \frac{dy}{y+1} &= (x+1)\, dx \\ \int \frac{dy}{y+1} &= \int (x+1)\, dx \\ \ln|y+1| &= \frac{1}{2}x^2 + x + C \\ y + 1 &= \pm e^{\frac{1}{2}x^2 + x + C} \\ y + 1 &= Ae^{\frac{1}{2}x(x+2)} \end{aligned}$$

Therefore,

$$y(x) = Ae^{\frac{1}{2}x(x+2)} - 1.$$