Problem 1.31

Solve the following differential equations:

(*d*) $yy'' = 2(y')^2$;

Solution

Subtract $(y')^2$ from both sides.

$$yy'' - (y')^2 = (y')^2$$

Divide both sides by $(y')^2$.

$$\frac{yy'' - (y')^2}{(y')^2} = 1$$

Recognize that the left side is the derivative of a quotient.

$$\frac{d}{dx}\left(-\frac{y}{y'}\right) = 1$$

Integrate both sides with respect to x.

$$-\frac{y}{y'} = x + C_1$$

Multiply both sides by -1.

$$\frac{y}{y'} = -(x + C_1)$$

Invert both sides.

$$\frac{y'}{y} = -\frac{1}{x + C_1}$$

This ODE can be solved with separation of variables.

$$\frac{dy}{y} = -\frac{dx}{x + C_1}$$

Integrate both sides.

$$\ln|y| = -\ln|x + C_1| + C_2$$

Exponentiate both sides.

$$e^{\ln|y|} = e^{\ln|x+C_1|^{-1}+C_2}$$

$$|y| = \frac{e^{C_2}}{|x + C_1|}$$

Remove the absolute value sign on the left by introducing $\pm$ on the right side.

$$y(x) = \frac{\pm e^{C_2}}{|x + C_1|}$$

Use new arbitrary constants on the right side, A and B, and drop the absolute value sign—we can do this because A is arbitrary. Therefore,

$$y(x) = \frac{A}{x + B}.$$