Problem 1.31

Solve the following differential equations:

(k) $2xyy' + y^2 - x^2 = 0$;

Solution

Rewrite the term with the derivative as follows.

$$x\frac{d}{dx}(y^2) + y^2 - x^2 = 0$$

Bring the x^2 term to the right.

$$x\frac{d}{dx}(y^2) + y^2 = x^2$$

Notice that the left side is exact and can be written as $d/dx(xy^2)$ as a result of the product rule.

$$\frac{d}{dx}(xy^2) = x^2$$

Integrate both sides with respect to x.

$$xy^2 = \frac{1}{3}x^3 + C$$

Divide both sides by x.

$$y^2 = \frac{1}{3}x^2 + \frac{C}{x}$$

Therefore,

$$y(x) = \pm\sqrt{\frac{1}{3}x^2 + \frac{C}{x}}.$$