Problem 1.31

Solve the following differential equations:

(w) $(x + y^2x)y' + x^2y^3 = 0$ $[y(1) = 1]$;

Solution

This ODE can be solved by separation of variables.

$$x(1 + y^2)\frac{dy}{dx} + x^2y^3 = 0$$

Bring x^2y^3 over to the right.

$$x(1 + y^2)\frac{dy}{dx} = -x^2y^3$$

Separate variables.

$$\frac{1 + y^2}{y^3}\,dy = -x\,dx$$

Integrate both sides.

$$\int^y \left(s^{-3} + \frac{1}{s}\right) ds = -\frac{1}{2}x^2 + C$$

Evaluate the integral on the left.

$$\frac{1}{-2}y^{-2} + \ln|y| = -\frac{1}{2}x^2 + C$$

Use the given boundary condition, $y(1) = 1$, to determine C.

$$-\frac{1}{2} = -\frac{1}{2} + C \quad \rightarrow \quad C = 0$$

So we have

$$\frac{1}{2}\left(x^2 - \frac{1}{y^2}\right) + \ln|y| = 0.$$

In order to obtain a single positive value of y when $x = 1$, we restrict the solution to positive values of y by dropping the absolute value sign.

$$\frac{1}{2}\left(x^2 - \frac{1}{y^2}\right) + \ln\, y = 0$$

Do note, though, that because we divided by x when we separated variables, the solution for y is not defined when $x = 0$. Therefore,

$$\frac{1}{2}\left(x^2 - \frac{1}{y^2}\right) + \ln\, y = 0, \quad x \neq 0.$$

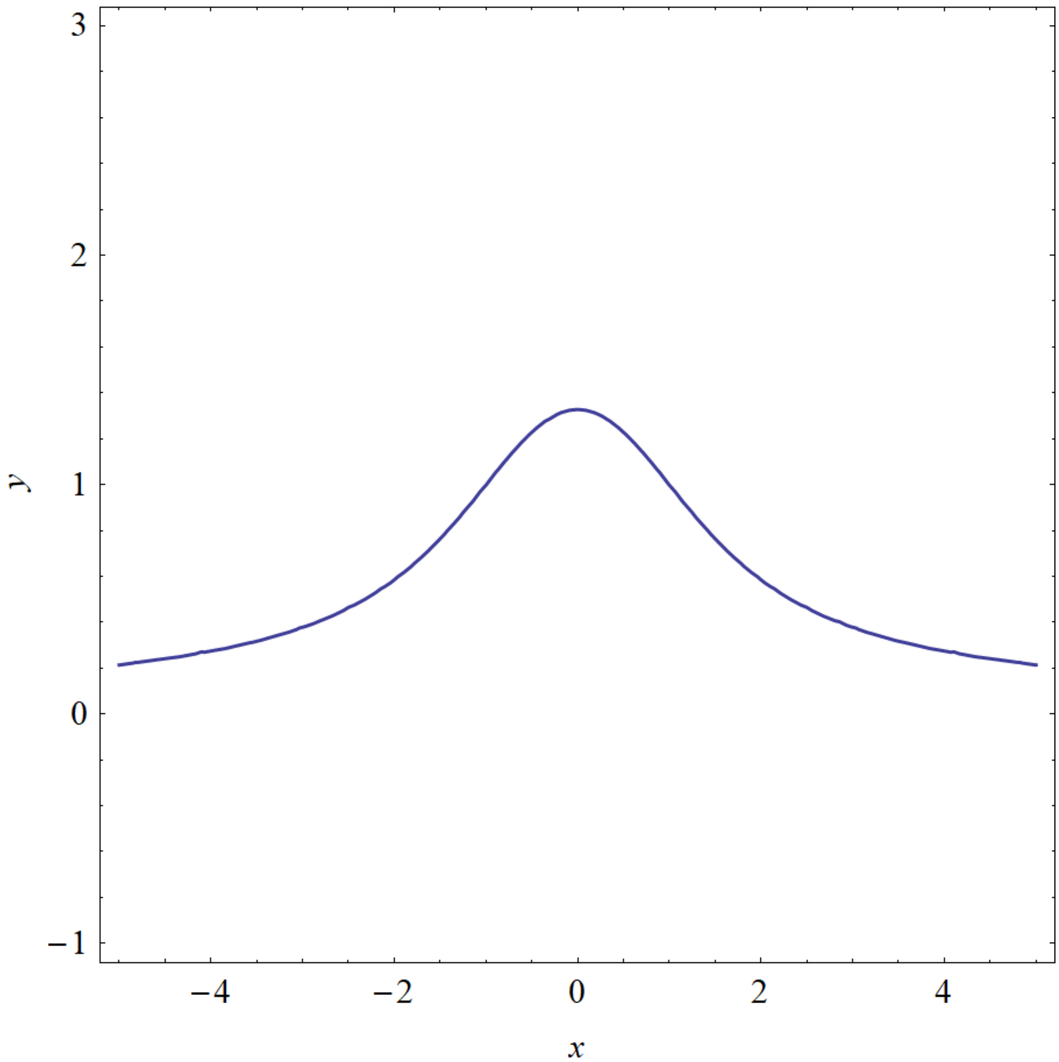

Figure 1: Plot of the solution for $-4 < x < 4$ and $-1 < y < 3$. There is a point of discontinuity in the graph when $x = 0$.