Problem 1

In each of Problems 1 through 6, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of $y$ as $t \to \infty$. If this behavior depends on the initial value of $y$ at $t = 0$, describe the dependency.

$$y' = 3 - 2y$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, 3 - 2y \rangle dt$$

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to converge to $y = 1.5$ as $t \to \infty$. 

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The (stable) equilibrium solution is found by setting \( y' = 0 \) in the differential equation and solving the resulting equation for \( y \).

\[
0 = 3 - 2y \\
y = \frac{3}{2}
\]