Problem 5

In each of Problems 1 through 6, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of $y$ as $t \to \infty$. If this behavior depends on the initial value of $y$ at $t = 0$, describe the dependency.

\[ y' = 1 + 2y \]

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

\[
\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, 1 + 2y \rangle dt
\]

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to diverge from $y = -0.5$ as $t \to \infty$. 

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The (unstable) equilibrium solution is found by setting $y' = 0$ in the differential equation and solving the resulting equation for $y$.

\begin{align*}
0 &= 1 + 2y \\
y &= -\frac{1}{2}
\end{align*}