Problem 9

In each of Problems 7 through 10, write down a differential equation of the form $dy/dt = ay + b$ whose solutions have the required behavior as $t \to \infty$.

All other solutions diverge from $y = 2$.

Solution

dy/dt represents the rate of change of y with respect to t. Setting $dy/dt = 0$ in the differential equation allows us to find the equilibrium solution.

$$0 = ay + b$$

$$y = -\frac{b}{a}$$

a and b need to be chosen so that this ratio evaluates to 2. In addition, a has to be positive so that the nonequilibrium solutions diverge as $t \to \infty$. One possible choice is $a = 1$ and $b = -2$.

$$\frac{dy}{dt} = y - 2$$

Below in Figure 1 is the direction field for this differential equation along with possible solutions.

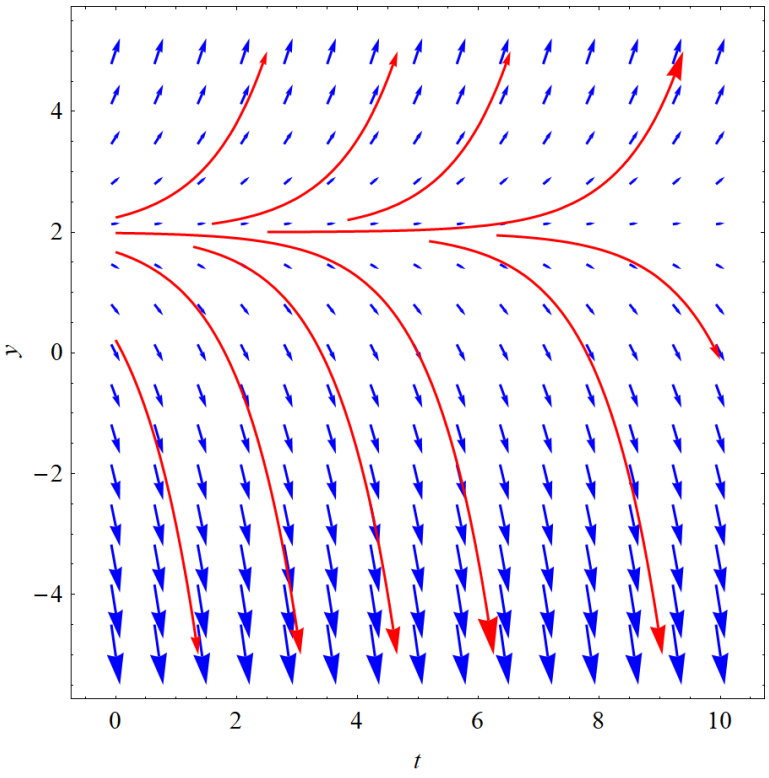

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to diverge from $y = 2$ as $t \to \infty$.