Problem 9

In each of Problems 7 through 10, write down a differential equation of the form \( \frac{dy}{dt} = ay + b \) whose solutions have the required behavior as \( t \to \infty \).

All other solutions diverge from \( y = 2 \).

Solution

\( \frac{dy}{dt} \) represents the rate of change of \( y \) with respect to \( t \). Setting \( \frac{dy}{dt} = 0 \) in the differential equation allows us to find the equilibrium solution.

\[
0 = ay + b \\
y = -\frac{b}{a}
\]

\( a \) and \( b \) need to be chosen so that this ratio evaluates to 2. In addition, \( a \) has to be positive so that the nonequilibrium solutions diverge as \( t \to \infty \). One possible choice is \( a = 1 \) and \( b = -2 \).

\[
\frac{dy}{dt} = y - 2
\]

Below in Figure 1 is the direction field for this differential equation along with possible solutions.

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to diverge from \( y = 2 \) as \( t \to \infty \).