Problem 14

In each of Problems 11 through 14, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that in these problems the equations are not of the form $y' = ay + b$, and the behavior of their solutions is somewhat more complicated than for the equations in the text.

$$y' = y(y-2)^2$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, y(y-2)^2 \rangle \, dt$$

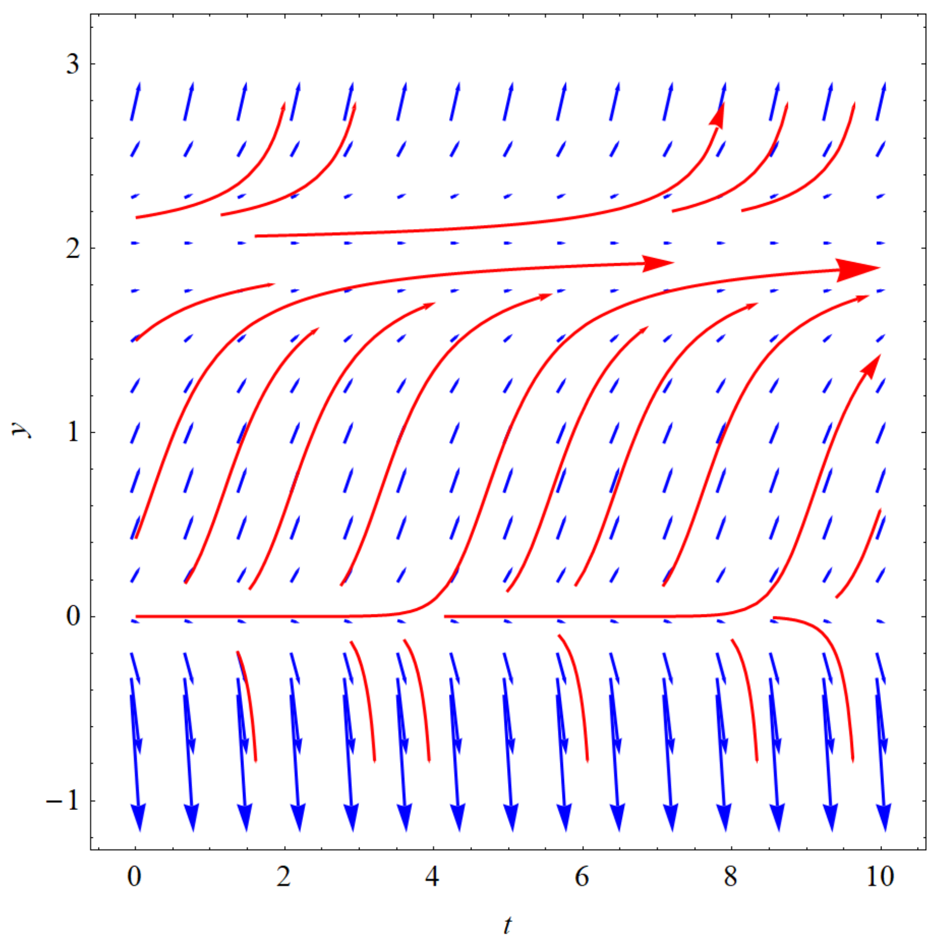

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is.

The equilibrium solution is found by setting $y' = 0$ in the differential equation and solving the resulting equation for y.

$$0 = y(y-2)^2$$

$$y = 0 \quad \text{or} \quad y = 2$$

Notice from Figure 1 that the behavior of y as $t \to \infty$ depends on what the initial condition is. If $y(t=0) > 2$, then the solution diverges to ∞ as $t \to \infty$. If $0 < y(t=0) < 2$, then the solution converges to $y = 2$ as $t \to \infty$. If $y(t=0) < 0$, then the solution diverges to $-\infty$ as $t \to \infty$.