Problem 15

Consider the following list of differential equations, some of which produced the direction fields shown in Figures 1.1.5 through 1.1.10. In each of Problems 15 through 20 identify the differential equation that corresponds to the given direction field.

(a) \( y' = 2y - 1 \)  \hspace{1cm} (b) \( y' = 2 + y \)  \hspace{1cm} (c) \( y' = y - 2 \)

(d) \( y' = y(y + 3) \)  \hspace{1cm} (e) \( y' = y(y - 3) \)  \hspace{1cm} (f) \( y' = 1 + 2y \)

(g) \( y' = -2 - y \)  \hspace{1cm} (h) \( y' = y(3 - y) \)  \hspace{1cm} (i) \( y' = 1 - 2y \)

(j) \( y' = 2 - y \)

The direction field of Figure 1.1.5.

\[ \text{FIGURE 1.1.5} \quad \text{Problem 15.} \]

Solution

Notice from the direction field that \( y = 2 \) is an equilibrium solution. This means that if we set \( y' = 0 \) in the differential equation, the solution to the resulting equation is \( y = 2 \). Consequently, the correct answer is either (c) or (j). Also, notice from the direction field that \( y' \) is negative if \( y > 2 \). Therefore, the differential equation in (j) corresponds with this direction field.

\[ y' = 2 - y \]
Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The solution curves are drawn (from left to right) such that the direction field vectors are tangent to them at every point.