Problem 23

Newton's law of cooling states that the temperature of an object changes at a rate proportional to the difference between the temperature of the object itself and the temperature of its surroundings (the ambient air temperature in most cases). Suppose that the ambient temperature is 70°F and that the rate constant is 0.05 $(\text{min})^{-1}$. Write a differential equation for the temperature of the object at any time. Note that the differential equation is the same whether the temperature of the object is above or below the ambient temperature.

Solution

According to Newton's law of cooling,

$$\underbrace{\frac{dT}{dt}}_{\text{rate of temperature change}} \quad \underbrace{\propto}_{\text{is proportional to}} \quad \underbrace{T - T_\infty}_{\text{temperature difference}},$$

where T_∞ is the ambient temperature. In order to change this proportionality to an equation, introduce the positive proportionality constant k.

$$\frac{dT}{dt} = -k(T - T_\infty)$$

The minus sign is included because if the object temperature is higher than the ambient temperature ($T - T_\infty > 0$), we expect the object to cool off ($dT/dt < 0$). Therefore, using the provided constants,

$$\frac{dT}{dt} = -0.05(T - 70).$$