Problem 26

In each of Problems 26 through 33, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that the right sides of these equations depend on t as well as y; therefore, their solutions can exhibit more complicated behavior than those in the text.

$$y' = -2 + t - y$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, -2 + t - y \rangle \, dt$$

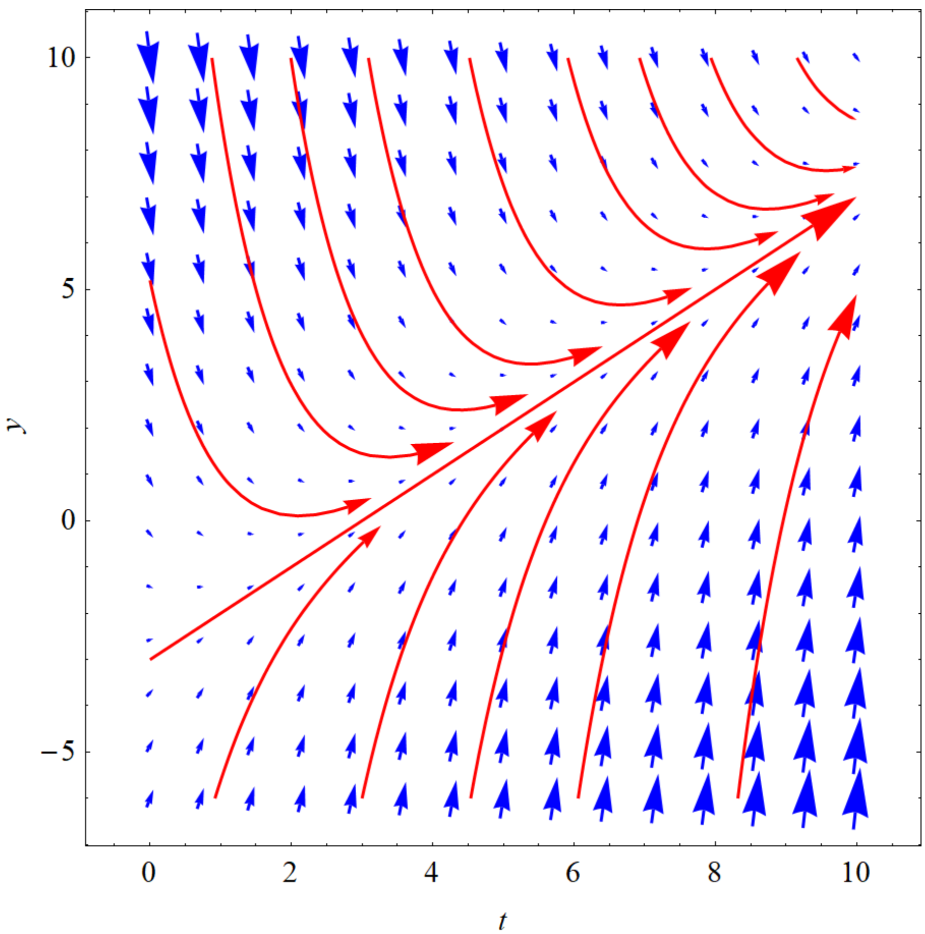

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The nonequilibrium solutions appear to converge to the straight red curve, which has a slope of 1, as $t \to \infty$ no matter what the initial condition is.

The asymptotic solution is found by setting $y' = 1$ in the differential equation and solving the resulting equation for y.

$$1 = -2 + t - y$$

$$y = t - 3$$