Problem 29

In each of Problems 26 through 33, draw a direction field for the given differential equation. Based on the direction field, determine the behavior of y as $t \to \infty$. If this behavior depends on the initial value of y at $t = 0$, describe this dependency. Note that the right sides of these equations depend on t as well as y; therefore, their solutions can exhibit more complicated behavior than those in the text.

$$y' = t + 2y$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, t + 2y \rangle\, dt$$

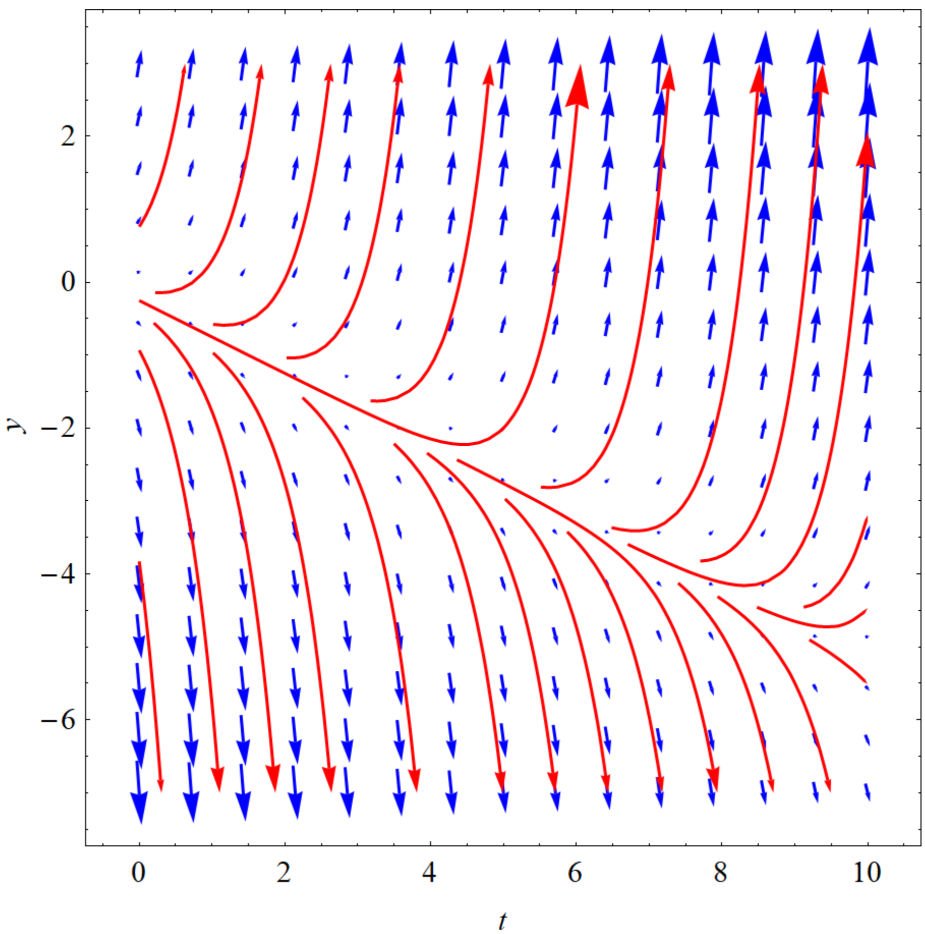

Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. The solutions appear to diverge from a straight line, which has slope $-1/2$, as $t \to \infty$.

The asymptotic solution is found by setting $y' = -1/2$ in the differential equation and solving the resulting equation for y.

$$-\frac{1}{2} = t + 2y$$

$$y = -\frac{1}{2}\left(t + \frac{1}{2}\right)$$

If the initial condition is below this line, then the solution diverges to $-\infty$, and if the initial condition is above the line, then the solution diverges to ∞.