Problem 11

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

\[ 2t^2 y'' + 3ty' - y = 0, \quad t > 0; \quad y_1(t) = t^{1/2}, \quad y_2(t) = t^{-1} \]

Solution

\[ 2t^2 y'' + 3ty' - y_1' = 0 \]

\[ 2t^2 \frac{d^2}{dt^2} (t^{1/2}) + 3t \frac{d}{dt} (t^{1/2}) - t^{1/2} = 0 \]

\[ 2t^2 \left( \frac{1}{2} \right) \left( -\frac{1}{2} \right) t^{-3/2} + 3t \left( \frac{1}{2} \right) t^{-1/2} - t^{1/2} = 0 \]

\[ -\frac{1}{2} t^{1/2} + \frac{3}{2} t^{1/2} - t^{1/2} = 0 \]

\[ 0 = 0 \]

The first solution is verified.

\[ 2t^2 y_2'' + 3ty_2' - y_2 = 0 \]

\[ 2t^2 \frac{d^2}{dt^2} (t^{-1}) + 3t \frac{d}{dt} (t^{-1}) - t^{-1} = 0 \]

\[ 2t^2 (-1)(-2)t^{-3} + 3t(-1)t^{-2} - t^{-1} = 0 \]

\[ 4t^{-1} - 3t^{-1} - t^{-1} = 0 \]

\[ 0 = 0 \]

The second solution is verified.