Problem 12

In each of Problems 7 through 14, verify that each given function is a solution of the differential equation.

\[ t^2 y'' + 5ty' + 4y = 0, \quad t > 0; \quad y_1(t) = t^{-2}, \quad y_2(t) = t^{-2} \ln t \]

Solution

\[ t^2 y_1'' + 5ty_1' + 4y_1 = 0 \]
\[ t^2 \frac{d^2}{dt^2} (t^{-2}) + 5t \frac{d}{dt} (t^{-2}) + 4(t^{-2}) = 0 \]
\[ t^2 (-2) (-3)t^{-4} + 5t(-2)t^{-3} + 4t^{-2} = 0 \]
\[ 6t^{-2} - 10t^{-2} + 4t^{-2} = 0 \]
\[ 0 = 0 \]

The first solution is verified.

\[ t^2 y_2'' + 5ty_2' + 4y_2 = 0 \]
\[ t^2 \frac{d^2}{dt^2} (t^{-2} \ln t) + 5t \frac{d}{dt} (t^{-2} \ln t) + 4(t^{-2} \ln t) = 0 \]
\[ t^2 (6t^{-4} \ln t - 2t^{-4} - 3t^{-4}) + 5t(-2t^{-3} \ln t + t^{-3}) + 4t^{-2} \ln t = 0 \]
\[ 6t^{-2} \ln t - 5t^{-2} - 10t^{-2} \ln t + 5t^{-2} + 4t^{-2} \ln t = 0 \]
\[ 0 = 0 \]

The second solution is verified.