Problem 18

In each of Problems 15 through 18, determine the values of \( r \) for which the given differential equation has solutions of the form \( y = e^{rt} \).

\[
y''' - 3y'' + 2y' = 0
\]

Solution

Because the terms on the left have constant coefficients, the solution to the ODE is of the form \( y = e^{rt} \). Substitute it into the equation to determine \( r \).

\[
\frac{d^3}{dt^3}(e^{rt}) - 3 \frac{d^2}{dt^2}(e^{rt}) + 2 \frac{d}{dt}(e^{rt}) = 0
\]

\[
r^3 e^{rt} - 3r^2 e^{rt} + 2re^{rt} = 0
\]

Divide both sides by \( e^{rt} \).

\[
r^3 - 3r^2 + 2r = 0
\]

\[
r(r - 1)(r - 2) = 0
\]

\[
r = 0 \quad \text{or} \quad r = 1 \quad \text{or} \quad r = 2
\]

Therefore, 1 and \( e^t \) and \( e^{2t} \) are three solutions to the ODE. The general solution is

\[
y(t) = C_1 + C_2 e^t + C_3 e^{2t},
\]

where \( C_1 \) and \( C_2 \) and \( C_3 \) are arbitrary constants.