Problem 25

In each of Problems 25 through 28, verify that each given function is a solution of the given partial differential equation.

\[ u_{xx} + u_{yy} = 0; \quad u_1(x, y) = \cos x \cosh y, \quad u_2(x, y) = \ln(x^2 + y^2) \]

Solution

\[ \frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} = 0 \]

\[ \frac{\partial^2}{\partial x^2} (\cos x \cosh y) + \frac{\partial^2}{\partial y^2} (\cos x \cosh y) \approx 0 \]

\[ -\cos x \cosh y + \cos x \cosh y \approx 0 \]

\[ 0 = 0 \]

The first solution is verified.

\[ \frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} = 0 \]

\[ \frac{\partial^2}{\partial x^2} [\ln(x^2 + y^2)] + \frac{\partial^2}{\partial y^2} [\ln(x^2 + y^2)] \approx 0 \]

\[ \frac{\partial}{\partial x} \left( \frac{2x}{x^2 + y^2} \right) + \frac{\partial}{\partial y} \left( \frac{-2y}{x^2 + y^2} \right) \approx 0 \]

\[ \frac{2(x^2 + y^2) - 2x(2x)}{(x^2 + y^2)^2} + \frac{2(x^2 + y^2) - 2y(2y)}{(x^2 + y^2)^2} \approx 0 \]

\[ \frac{-2x^2 + 2y^2}{(x^2 + y^2)^2} + \frac{2x^2 - 2y^2}{(x^2 + y^2)^2} \approx 0 \]

\[ -2x^2 + 2y^2 - 2x^2 + 2y^2 \approx 0 \]

\[ 0 = 0 \]

The second solution is verified.