Problem 25

In each of Problems 25 through 28, verify that each given function is a solution of the given partial differential equation.

$$u_{xx} + u_{yy} = 0; \qquad u_1(x,y) = \cos x \cosh y, \quad u_2(x,y) = \ln(x^2+y^2)$$

Solution

$$\begin{aligned}
\frac{\partial^2 u_1}{\partial x^2} + \frac{\partial^2 u_1}{\partial y^2} &\stackrel{?}{=} 0 \\
\frac{\partial^2}{\partial x^2}(\cos x \cosh y) + \frac{\partial^2}{\partial y^2}(\cos x \cosh y) &\stackrel{?}{=} 0 \\
-\cos x \cosh y + \cos x \cosh y &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}$$

The first solution is verified.

$$\begin{aligned}
\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial y^2} &\stackrel{?}{=} 0 \\
\frac{\partial^2}{\partial x^2}[\ln(x^2+y^2)] + \frac{\partial^2}{\partial y^2}[\ln(x^2+y^2)] &\stackrel{?}{=} 0 \\
\frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2}\right) + \frac{\partial}{\partial y}\left(\frac{2y}{x^2+y^2}\right) &\stackrel{?}{=} 0 \\
\frac{2(x^2+y^2)-2x(2x)}{(x^2+y^2)^2} + \frac{2(x^2+y^2)-2y(2y)}{(x^2+y^2)^2} &\stackrel{?}{=} 0 \\
\frac{-2x^2+2y^2}{(x^2+y^2)^2} + \frac{2x^2-2y^2}{(x^2+y^2)^2} &\stackrel{?}{=} 0 \\
\frac{-2x^2+2y^2}{(x^2+y^2)^2} - \frac{-2x^2+2y^2}{(x^2+y^2)^2} &\stackrel{?}{=} 0 \\
0 &= 0
\end{aligned}$$

The second solution is verified.