Problem 27

In each of Problems 25 through 28, verify that each given function is a solution of the given partial differential equation.

\[ a^2 u_{xx} = u_{tt}; \quad u_1(x, t) = \sin \lambda x \sin \lambda at, \quad u_2(x, t) = \sin(x - at), \quad \lambda \text{ a real constant} \]

Solution

\[ a^2 \frac{\partial^2 u_1}{\partial x^2} = \frac{\partial^2 u_1}{\partial t^2} \]
\[ a^2 \frac{\partial^2}{\partial x^2} (\sin \lambda x \sin \lambda at) = \frac{\partial^2}{\partial t^2} (\sin \lambda x \sin \lambda at) \]
\[ a^2 (\varepsilon \lambda^2 \sin \lambda x) \sin \lambda at = \sin \lambda x (-\lambda^2 a^2 \sin \lambda at) \]
\[ -a^2 \lambda^2 \sin \lambda x \sin \lambda at = -a^2 \lambda^2 \sin \lambda x \sin \lambda at \]

The first solution is verified.

\[ a^2 \frac{\partial^2 u_2}{\partial x^2} = \frac{\partial^2 u_2}{\partial t^2} \]
\[ a^2 \frac{\partial^2}{\partial x^2} [\sin(x - at)] = \frac{\partial^2}{\partial t^2} [\sin(x - at)] \]
\[ a^2 [\varepsilon \sin(x - at)] = (-a)^2 [\varepsilon \sin(x - at)] \]
\[ -a^2 \sin(x - at) = -a^2 \sin(x - at) \]

The second solution is verified.