Problem 27

In each of Problems 25 through 28, verify that each given function is a solution of the given partial differential equation.

$$a^2 u_{xx} = u_{tt}; \qquad u_1(x,t) = \sin\lambda x \sin\lambda at, \quad u_2(x,t) = \sin(x-at), \quad \lambda \text{ a real constant}$$

Solution

$$\begin{aligned}
a^2\frac{\partial^2 u_1}{\partial x^2} &\stackrel{?}{=} \frac{\partial^2 u_1}{\partial t^2} \\
a^2\frac{\partial^2}{\partial x^2}(\sin\lambda x\sin\lambda at) &\stackrel{?}{=} \frac{\partial^2}{\partial t^2}(\sin\lambda x\sin\lambda at) \\
a^2(-\lambda^2\sin\lambda x)\sin\lambda at &\stackrel{?}{=} \sin\lambda x(-\lambda^2 a^2\sin\lambda at) \\
-a^2\lambda^2\sin\lambda x\sin\lambda at &= -a^2\lambda^2\sin\lambda x\sin\lambda at
\end{aligned}$$

The first solution is verified.

$$\begin{aligned}
a^2\frac{\partial^2 u_2}{\partial x^2} &\stackrel{?}{=} \frac{\partial^2 u_2}{\partial t^2} \\
a^2\frac{\partial^2}{\partial x^2}[\sin(x-at)] &\stackrel{?}{=} \frac{\partial^2}{\partial t^2}[\sin(x-at)] \\
a^2[-\sin(x-at)] &\stackrel{?}{=} (-a)^2[-\sin(x-at)] \\
-a^2\sin(x-at) &= -a^2\sin(x-at)
\end{aligned}$$

The second solution is verified.