

Problem 5

In each of Problems 1 through 12:

- Draw a direction field for the given differential equation.
- Based on an inspection of the direction field, describe how solutions behave for large t .
- Find the general solution of the given differential equation, and use it to determine how solutions behave as $t \rightarrow \infty$.

$$y' - 2y = 3e^t$$

Solution

The direction field is a two-dimensional vector field that shows what the direction of the solution is at every point in a region. Every solution to the differential equation is a curve drawn such that the direction field vectors are tangent to it at every point.

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, 2y + 3e^t \rangle dt$$

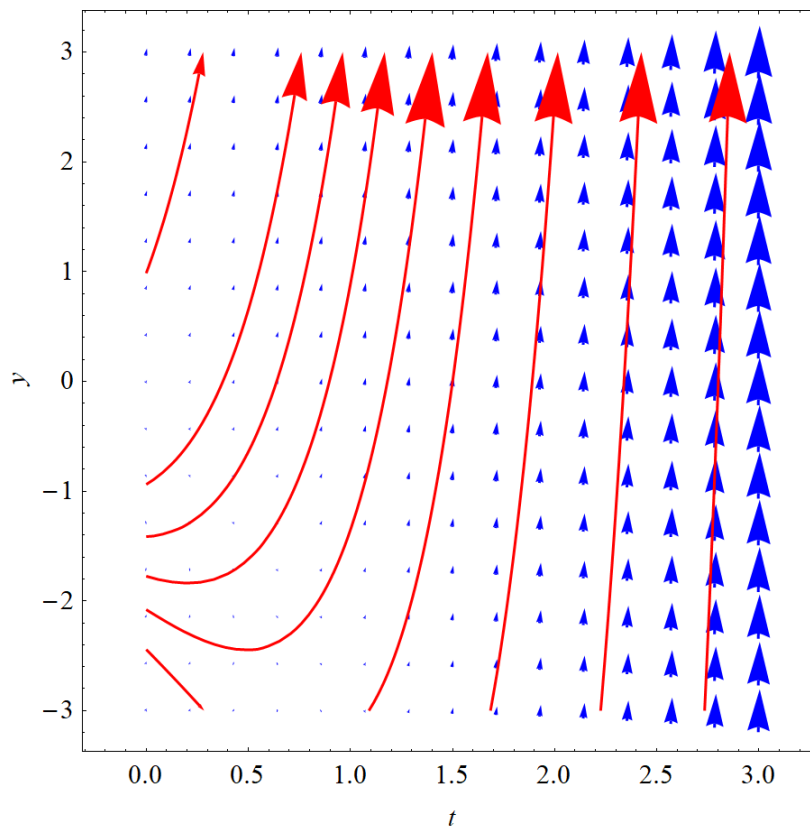


Figure 1: In blue are the direction field vectors and in red are possible solutions to the differential equation, depending what the initial condition is. All solutions appear to diverge to ∞ as $t \rightarrow \infty$.

$$y' - 2y = 3e^t$$

This is a first-order linear inhomogeneous equation, so it can be solved with an integrating factor.

$$I(t) = \exp \left[\int^t (-2) ds \right] = e^{-2t}$$

Multiply both sides of the ODE by $I(t)$.

$$e^{-2t}y' - 2e^{-2t}y = 3e^{-t}$$

The left side can be written as $d/dt(Iy)$ by the product rule.

$$\frac{d}{dt}(e^{-2t}y) = 3e^{-t}$$

Integrate both sides with respect to t .

$$e^{-2t}y = -3e^{-t} + C$$

Multiply both sides by e^{2t} to solve for $y(t)$.

$$y(t) = -3e^t + Ce^{2t}$$

The solution diverges to ∞ as $t \rightarrow \infty$.