Problem 17

In each of Problems 13 through 20, find the solution of the given initial value problem.

$$y' - 2y = e^{2t}, \qquad y(0) = 2$$

Solution

This is a first-order linear inhomogeneous equation, so it can be solved with an integrating factor.

$$I(t) = \exp\left[\int^t (-2)\, ds\right] = e^{-2t}$$

Multiply both sides of the ODE by $I(t)$.

$$e^{-2t}y' - 2e^{-2t}y = 1$$

The left side can be written as $d/dt(Iy)$ by the product rule.

$$\frac{d}{dt}(e^{-2t}y) = 1$$

Integrate both sides with respect to t.

$$e^{-2t}y = t + C$$

Multiply both sides by e^{2t} to solve for $y(t)$.

$$y(t) = e^{2t}(t + C)$$

Apply the initial condition $y(0) = 2$ here to determine C.

$$y(0) = e^0(0 + C) = 2 \quad \rightarrow \quad C = 2$$

Therefore,

$$y(t) = e^{2t}(t + 2).$$