Problem 34

In each of Problems 34 through 37, construct a first order linear differential equation whose solutions have the required behavior as \( t \to \infty \). Then solve your equation and confirm that the solutions do indeed have the specified property.

All solutions have the limit 3 as \( t \to \infty \).

Solution

The rate of change of \( y \) will become negligible as \( t \) gets big enough, so we choose

\[
y' + y = 3.
\]

This is a first-order linear inhomogeneous ODE, so it can be solved by multiplying both sides by an integrating factor \( I \).

\[
I = \exp \left( \int 1 \, ds \right) = e^t
\]

Proceed with the multiplication.

\[
e^t y' + e^t y = 3 e^t
\]

The left side can be written as \( \frac{d}{dt}(e^t y) \) using the product rule.

\[
\frac{d}{dt}(e^t y) = 3 e^t
\]

Integrate both sides with respect to \( t \).

\[
e^t y = \int e^s \, ds + C = 3 e^t + C
\]

Divide both sides by \( e^t \) to obtain the general solution for \( y \).

\[
y(t) = 3 + \frac{C}{e^t}
\]

Take the limit of both sides as \( t \to \infty \).

\[
\lim_{t \to \infty} y(t) = \lim_{t \to \infty} 3 + \lim_{t \to \infty} \frac{C}{e^t} = 3 + \lim_{t \to \infty} 0 = 3
\]

Therefore, all solutions of \( y' + y = 3 \) have the limit 3 as \( t \to \infty \).