Problem 4

In each of Problems 1 through 8, solve the given differential equation.

\[ y' = \frac{(3x^2 - 1)}{(3 + 2y)} \]

Solution

This ODE is separable because it is of the form \( y' = f(x)g(y) \), so it can be solved by separating variables.

\[ \frac{dy}{dx} = \frac{3x^2 - 1}{3 + 2y} \]

Bring the terms with \( y \) to the left and bring the terms with \( x \) to the right.

\[ (3 + 2y) \, dy = (3x^2 - 1) \, dx \]

Integrate both sides.

\[ \int (3 + 2y) \, dy = \int (3x^2 - 1) \, dx \]

\[ 3y + y^2 = x^3 - x + C \]

Now solve the equation for \( y \).

\[ y^2 + 3y - (x^3 - x + C) = 0 \]

\[ y(x) = \frac{-3 \pm \sqrt{9 + 4(x^3 - x + C)}}{2} \].