Problem 24

Solve the initial value problem

$$y' = (2 - e^x)/(3 + 2y), \qquad y(0) = 0$$

and determine where the solution attains its maximum value.

Solution

This ODE is separable because it is of the form $y' = f(x)g(y)$, so it can be solved by separating variables.

$$\frac{dy}{dx} = \frac{2 - e^x}{3 + 2y}$$

Bring the terms with y to the left and bring the terms with x to the right.

$$(3 + 2y)\, dy = (2 - e^x)\, dx$$

Integrate both sides.

$$3y + y^2 = 2x - e^x + C$$

Apply the initial condition now to determine C.

$$0 + 0 = 0 - 1 + C \quad \rightarrow \quad C = 1$$

As a result,

$$3y + y^2 = 2x - e^x + 1$$

$$y^2 + 3y - (2x - e^x + 1) = 0$$

$$y(x) = \frac{-3 \pm \sqrt{9 + 4(2x - e^x + 1)}}{2}$$

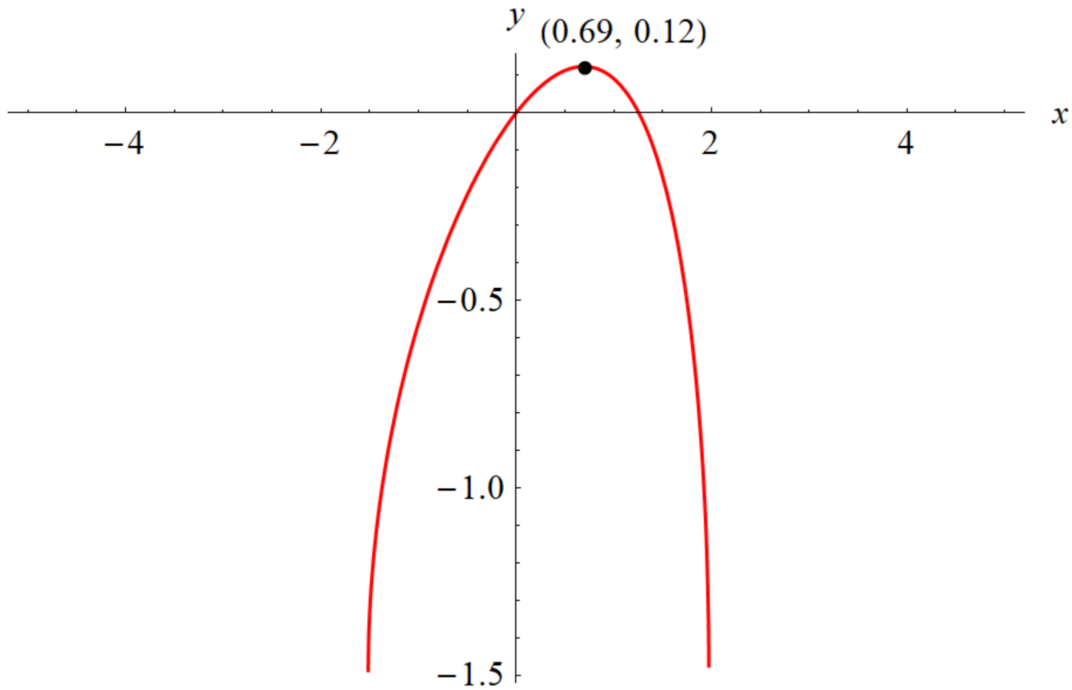
We choose the plus sign so that the initial condition is satisfied. Therefore,

$$y(x) = \frac{-3 + \sqrt{13 + 4(2x - e^x)}}{2}.$$

Inspecting the ODE, we see that $dy/dx = 0$ when $2 - e^x = 0$, or $x = \ln 2 \approx 0.69$. Plugging this value into the solution, we get

$$y(\ln 2) \approx 0.124.$$

Therefore, the solution attains its maximum value at approximately $(0.69, 0.124)$.


y
(0.69, 0.12)
x
−4
−2
2
4
−0.5
−1.0
−1.5