Problem 1

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

$$(t-3)y' + (\ln t)y = 2t, \qquad y(1) = 2$$

Solution

According to Theorem 2.4.1, a unique solution to

$$y' + p(t)y = g(t), \qquad y(t_0) = y_0$$

exists throughout any interval in t containing the point t_0 where the functions, $p(t)$ and $g(t)$, are continuous. Divide both sides of the ODE by $t-3$ to put it in standard form.

$$y' + \frac{\ln t}{t-3}y = \frac{2t}{t-3}$$

Both $p(t)$ and $g(t)$ in this case are discontinuous at $t = 3$, and the logarithm is defined for $0 < t < \infty$. Since $t_0 = 1 < 3$, a unique solution will exist for $0 < t < 3$.