Problem 1

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

\[(t - 3)y' + (\ln t)y = 2t, \quad y(1) = 2\]

Solution

According to Theorem 2.4.1, a unique solution to

\[y' + p(t)y = g(t), \quad y(t_0) = y_0\]

exists throughout any interval in \(t\) containing the point \(t_0\) where the functions, \(p(t)\) and \(g(t)\), are continuous. Divide both sides of the ODE by \(t - 3\) to put it in standard form.

\[y' + \frac{\ln t}{t - 3}y = \frac{2t}{t - 3}\]

Both \(p(t)\) and \(g(t)\) in this case are discontinuous at \(t = 3\), and the logarithm is defined for \(0 < t < \infty\). Since \(t_0 = 1 < 3\), a unique solution will exist for \(0 < t < 3\).