Problem 2

In each of Problems 1 through 6, determine (without solving the problem) an interval in which the solution of the given initial value problem is certain to exist.

\[ t(t - 4)y' + y = 0, \quad y(2) = 1 \]

Solution

According to Theorem 2.4.1, a unique solution to

\[ y' + p(t)y = g(t), \quad y(t_0) = y_0 \]

exists throughout any interval in \( t \) containing the point \( t_0 \) where the functions, \( p(t) \) and \( g(t) \), are continuous. Divide both sides of the ODE by \( t(t - 4) \) to put it in standard form.

\[ y' + \frac{1}{t(t - 4)}y = 0 \]

\( p(t) \) is discontinuous at \( t = 0 \) and \( t = 4 \). Since \( t_0 = 2 \) is between 0 and 4, a unique solution will exist for \( 0 < t < 4 \).