Problem 7

In each of Problems 7 through 12, state where in the ty-plane the hypotheses of Theorem 2.4.2 are satisfied.

$$y' = \frac{t-y}{2t+5y}$$

Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t,y), \qquad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that f and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains (t_0, y_0). In this exercise

$$f(t,y) = \frac{t-y}{2t+5y} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{(-1)(2t+5y)-(5)(t-y)}{(2t+5y)^2} = -\frac{7t}{(2t+5y)^2}.$$

Both f and $\partial f/\partial y$ are discontinuous when $2t+5y=0$, so the hypotheses of Theorem 2.4.2 are satisfied if $2t+5y<0$ or $2t+5y>0$.