Problem 10

In each of Problems 7 through 12, state where in the $ty$-plane the hypotheses of Theorem 2.4.2 are satisfied.

\[ y' = (t^2 + y^2)^{3/2} \]

Solution

According to Theorem 2.4.2, a unique solution to

\[ y' = f(t, y), \quad y(t_0) = y_0 \]

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that $f$ and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains $(t_0, y_0)$. In this exercise

\[ f(t, y) = (t^2 + y^2)^{3/2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{3}{2} (t^2 + y^2)^{1/2} (2y) = 3y(t^2 + y^2)^{1/2}. \]

$f$ is continuous everywhere, and $\partial f/\partial y$ is continuous everywhere. Therefore, the hypotheses of Theorem 2.4.2 are satisfied everywhere.