Problem 12

In each of Problems 7 through 12, state where in the ty-plane the hypotheses of Theorem 2.4.2 are satisfied.

$$\frac{dy}{dt} = \frac{(\cot t)y}{1+y}$$

Solution

According to Theorem 2.4.2, a unique solution to

$$y' = f(t, y), \qquad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that f and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains (t_0, y_0). In this exercise

$$f(t, y) = \frac{(\cot t)y}{1+y} = \frac{\cos t}{\sin t}\frac{y}{1+y} \quad \text{and} \quad \frac{\partial f}{\partial y} = (\cot t)\frac{(1)(1+y) - (y)(1)}{(1+y)^2} = \frac{\cos t}{\sin t}\frac{1}{(1+y)^2}.$$

Both f and $\partial f/\partial y$ are discontinuous at $y = -1$ and $t = n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$. Therefore, the hypotheses of Theorem 2.4.2 are satisfied if $y \neq -1$ and $t \neq n\pi$.