# Problem 13

In each of Problems 13 through 16, solve the given initial value problem and determine how the interval in which the solution exists depends on the initial value $y_0$.

$$y' = -4t/y, \qquad y(0) = y_0$$

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**Solution**

**Method Using the Chain Rule**

$$\frac{dy}{dt} = -\frac{4t}{y}$$

Multiply both sides by $2y$.

$$2y\frac{dy}{dt} = -8t$$

The left side can be written as $d/dt(y^2)$ by the chain rule.

$$\frac{d}{dt}(y^2) = -8t$$

Integrate both sides with respect to $t$.

$$y^2 = -4t^2 + C_1$$

Apply the initial condition now to determine $C_1$.

$$y_0^2 = C_1$$

So then the previous equation becomes

$$y^2 = -4t^2 + y_0^2.$$

Therefore,

$$y(t) = \pm\sqrt{y_0^2 - 4t^2}.$$

**Method By Separating Variables**

$$\frac{dy}{dt} = -\frac{4t}{y}$$

Solve the ODE by separating variables.

$$y\,dy = -4t\,dt$$

Integrate both sides.

$$\frac{1}{2}y^2 = -2t^2 + C_2$$

Multiply both sides by 2.

$$y^2 = -4t^2 + 2C_2$$

Apply the initial condition now to determine $C_2$.

$$y_0^2 = 2C_2$$

So then the previous equation becomes

$$y^2 = -4t^2 + y_0^2.$$

Therefore,

$$y(t) = \pm\sqrt{y_0^2 - 4t^2}.$$

According to Theorem 2.4.2, a unique solution to

$$y' = f(t, y), \qquad y(t_0) = y_0$$

exists in some interval $t_0 - h < t < t_0 + h$ within $\alpha < t < \beta$, provided that $f$ and $\partial f/\partial y$ are continuous in a rectangle $\alpha < t < \beta$, $\gamma < y < \delta$ that contains $(t_0, y_0)$. In this exercise

$$f(t, y) = -\frac{4t}{y} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{4t}{y^2}.$$

Both $f$ and $\partial f/\partial y$ are discontinuous at $y = 0$, so as long as $y_0 \neq 0$, there will be an interval of $t$ where a unique solution exists.