Problem 17

In each of Problems 17 through 20, draw a direction field and plot (or sketch) several solutions of the given differential equation. Describe how solutions appear to behave as t increases and how their behavior depends on the initial value y_0 when $t = 0$.

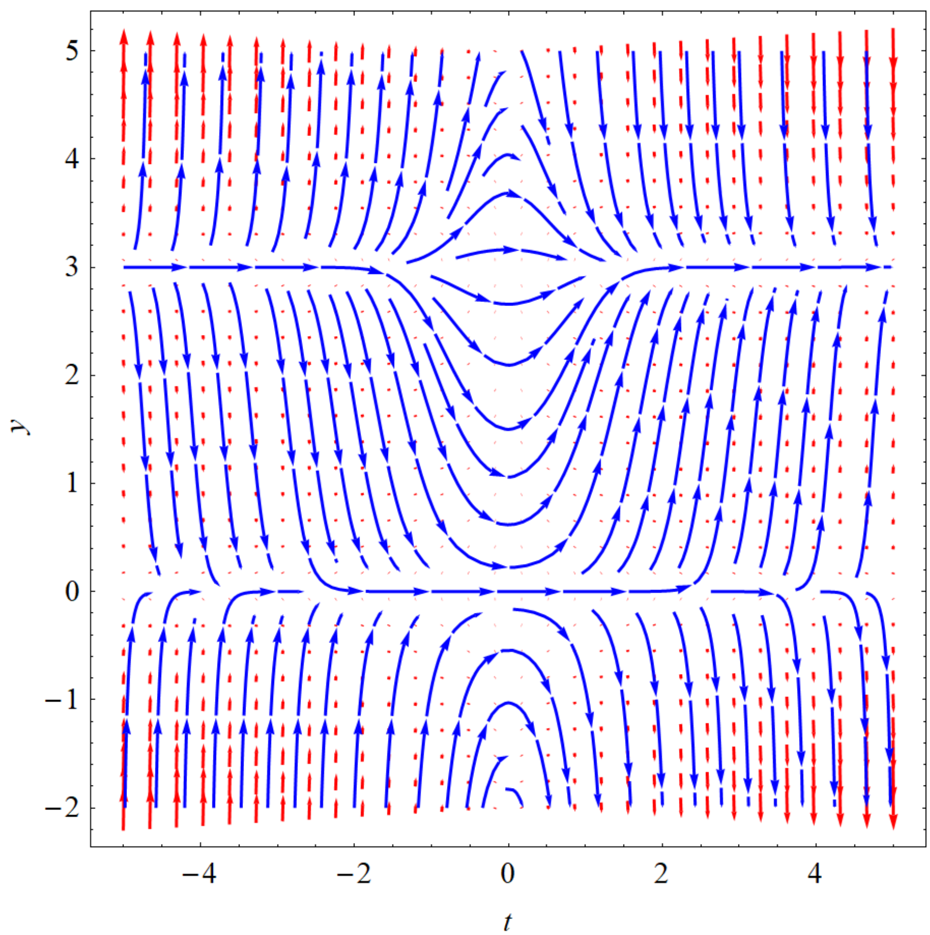
$$y' = ty(3 - y)$$

Solution

The direction field is the vector field

$$\langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, ty(3 - y) \rangle \, dt.$$

Below in red are the field vectors, and in blue are possible solution curves to the ODE, depending on the initial condition (t_0, y_0). The solution curves lie tangent to the field vectors at every point and never intersect.



Assuming an initial condition of the form $y(0) = y_0$, the solution will tend to $y = 3$ if $y_0 > 0$, tend to $y = 0$ if $y_0 = 0$, and tend to $y = -\infty$ if $y_0 < 0$.