Problem 20

In each of Problems 17 through 20, draw a direction field and plot (or sketch) several solutions of the given differential equation. Describe how solutions appear to behave as \( t \) increases and how their behavior depends on the initial value \( y_0 \) when \( t = 0 \).

\[ y' = t - 1 - y^2 \]

Solution

The direction field is the vector field

\[ \langle dt, dy \rangle = \left\langle 1, \frac{dy}{dt} \right\rangle dt = \langle 1, t - 1 - y^2 \rangle dt. \]

Below in red are the field vectors, and in blue are possible solution curves to the ODE, depending on the initial condition \((t_0, y_0)\). The solution curves lie tangent to the field vectors at every point and never intersect.

Assuming an initial condition of the form \( y(0) = y_0 \), the solution will tend to \( y = -\infty \) if \( y_0 \lesssim -0.02 \) and tend to \( y = \sqrt{t - 1} \) if \( y_0 \gtrsim -0.02 \).