

Problem 25

Let $y = y_1(t)$ be a solution of

$$y' + p(t)y = 0, \quad (\text{i})$$

and let $y = y_2(t)$ be a solution of

$$y' + p(t)y = g(t). \quad (\text{ii})$$

Show that $y = y_1(t) + y_2(t)$ is also a solution of Eq. (ii).

Solution

We have the following two equations to work with.

$$\begin{aligned} y_1' + p(t)y_1 &= 0 \\ y_2' + p(t)y_2 &= g(t) \end{aligned}$$

Add the respective sides of each equation.

$$y_1' + y_2' + p(t)y_1 + p(t)y_2 = g(t)$$

Factor $p(t)$ and the derivative.

$$(y_1 + y_2)' + p(t)(y_1 + y_2) = g(t)$$

Therefore, $y = y_1(t) + y_2(t)$ is also a solution of Eq. (ii).