Problem 1

Problems 1 through 6 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus $y$, determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the $ty$-plane.

$$\frac{dy}{dt} = ay + by^2, \quad a > 0, \quad b > 0, \quad y_0 \geq 0$$

Solution

In this problem $f(y) = ay + by^2$. For $a = 1$ and $b = 1$, the graph of $f(y)$ versus $y$ is shown below.

The equilibrium points are found by solving $f(y) = 0$ for $y$.

$$ay + by^2 = 0$$

$$y(a + by) = 0$$

$$y = \left\{ \frac{-a}{b}, 0 \right\}$$

Since $y_0 \geq 0$, only $y = 0$ is a relevant equilibrium point here. $f(y)$ is positive to the right of the equilibrium point, so $y = 0$ is unstable. It is represented by an open circle.

The arrows on the $y$-axis (phase line) indicate that $y$ is increasing in time.

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Some possible solution curves in the \(ty\)-plane for \(t \geq 0\) and \(y \geq 0\) are shown below. At every point, they are tangent to the direction field vectors \(⟨1, y + y^2⟩\).