

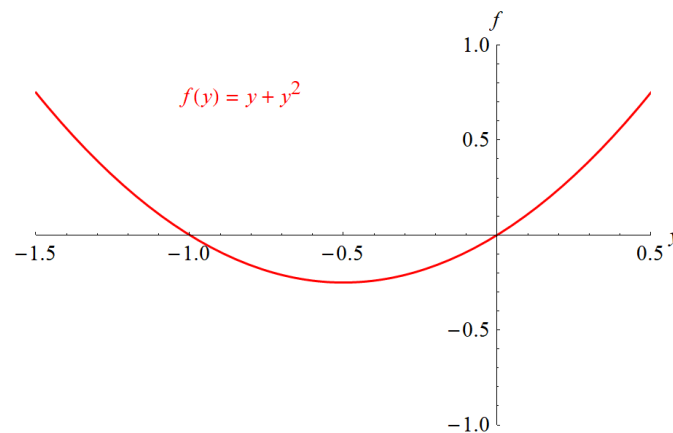
Problem 2

Problems 1 through 6 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus y , determine the critical (equilibrium) points, and classify each one as asymptotically stable or unstable. Draw the phase line, and sketch several graphs of solutions in the ty -plane.

$$dy/dt = ay + by^2, \quad a > 0, \quad b > 0, \quad -\infty < y_0 < \infty$$

Solution

In this problem $f(y) = ay + by^2$. For $a = 1$ and $b = 1$, the graph of $f(y)$ versus y is shown below.



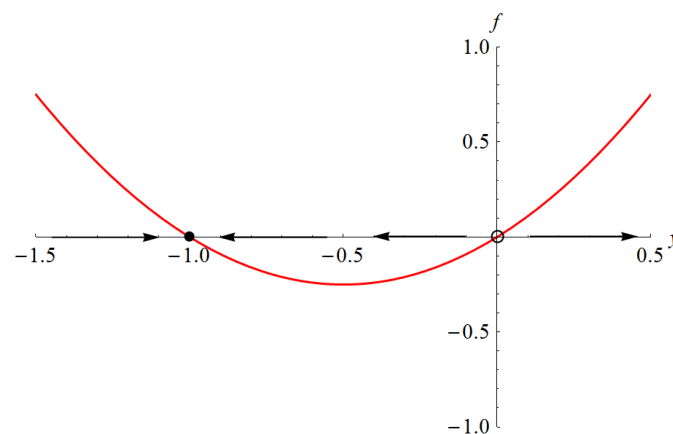
The equilibrium points are found by solving $f(y) = 0$ for y .

$$ay + by^2 = 0$$

$$y(a + by) = 0$$

$$y = \left\{ -\frac{a}{b}, 0 \right\}$$

As indicated below by the open and closed circles, $y = 0$ is unstable and $y = -a/b$ is stable.



The arrows pointing left and right on the y -axis (phase line) mean that y is decreasing and increasing in time, respectively.

Some possible solution curves in the ty -plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y + y^2 \rangle$.

