Problem 9

Problems 8 through 13 involve equations of the form \( \frac{dy}{dt} = f(y) \). In each problem sketch the graph of \( f(y) \) versus \( y \), determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable (see Problem 7). Draw the phase line, and sketch several graphs of solutions in the \( ty \)-plane.

\[
\frac{dy}{dt} = y^2(y^2 - 1), \quad -\infty < y_0 < \infty
\]

Solution

In this problem \( f(y) = y^2(y^2 - 1) \). Below is a graph of \( f(y) \) versus \( y \).

![Graph of f(y) vs y](image)

The equilibrium solutions are found by solving \( f(y) = 0 \) for \( y \).

\[
y^2(y^2 - 1) = 0
\]

\[
y^2 = 0 \quad \text{or} \quad y^2 - 1 = 0
\]

\[
y = \{-1, 0, 1\}
\]

As indicated below by the half-filled circle, \( y = 0 \) is stable from the right and unstable from the left. It is said to be semistable. \( y = -1 \) is stable, and \( y = 1 \) is unstable.

![Phase line](image)

The arrows pointing left and right on the \( y \)-axis (phase line) mean that \( y \) is decreasing and increasing in time, respectively.

www.stemjock.com
Some possible solution curves in the $ty$-plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $\langle 1, y^2(y^2 - 1) \rangle$. 