Problem 12

Problems 8 through 13 involve equations of the form $dy/dt = f(y)$. In each problem sketch the graph of $f(y)$ versus $y$, determine the critical (equilibrium) points, and classify each one asymptotically stable, unstable, or semistable (see Problem 7). Draw the phase line, and sketch several graphs of solutions in the $ty$-plane.

$$dy/dt = y^2(4 - y^2), \quad -\infty < y_0 < \infty$$

Solution

In this problem $f(y) = y^2(4 - y^2)$. Below is a graph of $f(y)$ versus $y$.

![Graph of f(y) versus y](image)

The equilibrium solutions are found by solving $f(y) = 0$ for $y$.

$$y^2(4 - y^2) = 0$$

$$y^2 = 0 \quad \text{or} \quad 4 - y^2 = 0$$

$$y = \{-2, 0, 2\}$$

As indicated below by the half-filled circle at $y = 0$, the equilibrium is stable from the left but unstable from the right. It is said to be semistable. $y = -2$ is an unstable equilibrium solution, and $y = 2$ is a stable equilibrium solution.

![Graph of f(y) versus y](image)

The arrows pointing left and right on the $y$-axis (phase line) mean that $y$ is decreasing and increasing in time, respectively.

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Some possible solution curves in the $ty$-plane for $t \geq 0$ are shown below. At every point, they are tangent to the direction field vectors $(1, y^2(4 - y^2))$. 

![Graph showing solution curves in the ty-plane](image-url)