Problem 1

In each of Problems 1 and 2, transform the given initial value problem into an equivalent problem with the initial point at the origin.

$$dy/dt = t^2 + y^2, \qquad y(1) = 2$$

Solution

The aim is to make the initial condition $y(0) = 0$ by changing variables. Start by changing the independent variable t.

$$t' = t - 1$$

This changes the initial condition $y(t = 1) = 2$ to $y(t' = 0) = 2$ and the ODE to

$$dy/dt = (t' + 1)^2 + y^2.$$

Use the chain rule to find what dy/dt is in terms of dy/dt'.

$$\frac{dy}{dt} = \frac{dy}{dt'}\frac{dt'}{dt} = \frac{dy}{dt'}(1) = \frac{dy}{dt'}$$

As a result of changing the independent variable, the initial value problem is

$$dy/dt' = (t' + 1)^2 + y^2, \qquad y(0) = 2.$$

Now change the dependent variable y.

$$y' = y - 2 \tag{1}$$

This changes the initial condition $y(t' = 0) = 2$ to $y'(t' = 0) = 0$ and the ODE to

$$dy/dt' = (t' + 1)^2 + (y' + 2)^2.$$

Differentiate both sides of equation (1) with respect to t' to find what dy/dt' is in terms of dy'/dt'.

$$\frac{dy'}{dt'} = \frac{dy}{dt'}$$

Therefore, the initial value problem can be written as

$$dy'/dt' = (t' + 1)^2 + (y' + 2)^2, \qquad y'(0) = 0.$$

Note that the primes here signify new variables and are not derivatives as usual.