Problem 1

In each of Problems 1 and 2, transform the given initial value problem into an equivalent problem with the initial point at the origin.

\[ \frac{dy}{dt} = t^2 + y^2, \quad y(1) = 2 \]

Solution

The aim is to make the initial condition \( y(0) = 0 \) by changing variables. Start by changing the independent variable \( t \).

\[ t' = t - 1 \]

This changes the initial condition \( y(t = 1) = 2 \) to \( y(t' = 0) = 2 \) and the ODE to

\[ \frac{dy}{dt} = (t' + 1)^2 + y^2. \]

Use the chain rule to find what \( \frac{dy}{dt} \) is in terms of \( \frac{dy}{dt'} \).

\[ \frac{dy}{dt} = \frac{dy}{dt'} \frac{dt'}{dt} = \frac{dy}{dt'} (1) = \frac{dy}{dt'} \]

As a result of changing the independent variable, the initial value problem is

\[ \frac{dy}{dt'} = (t' + 1)^2 + y^2, \quad y(0) = 2. \]

Now change the dependent variable \( y \).

\[ y' = y - 2 \quad (1) \]

This changes the initial condition \( y(t' = 0) = 2 \) to \( y'(t' = 0) = 0 \) and the ODE to

\[ \frac{dy}{dt'} = (t' + 1)^2 + (y' + 2)^2. \]

Differentiate both sides of equation (1) with respect to \( t' \) to find what \( \frac{dy}{dt'} \) is in terms of \( \frac{dy'}{dt'} \).

\[ \frac{dy'}{dt'} = \frac{dy}{dt} \]

Therefore, the initial value problem can be written as

\[ \frac{dy'}{dt'} = (t' + 1)^2 + (y' + 2)^2, \quad y'(0) = 0. \]

Note that the primes here signify new variables and are not derivatives as usual.