Problem 2

In each of Problems 1 through 6, solve the given difference equation in terms of the initial value y_0. Describe the behavior of the solution as $n \to \infty$.

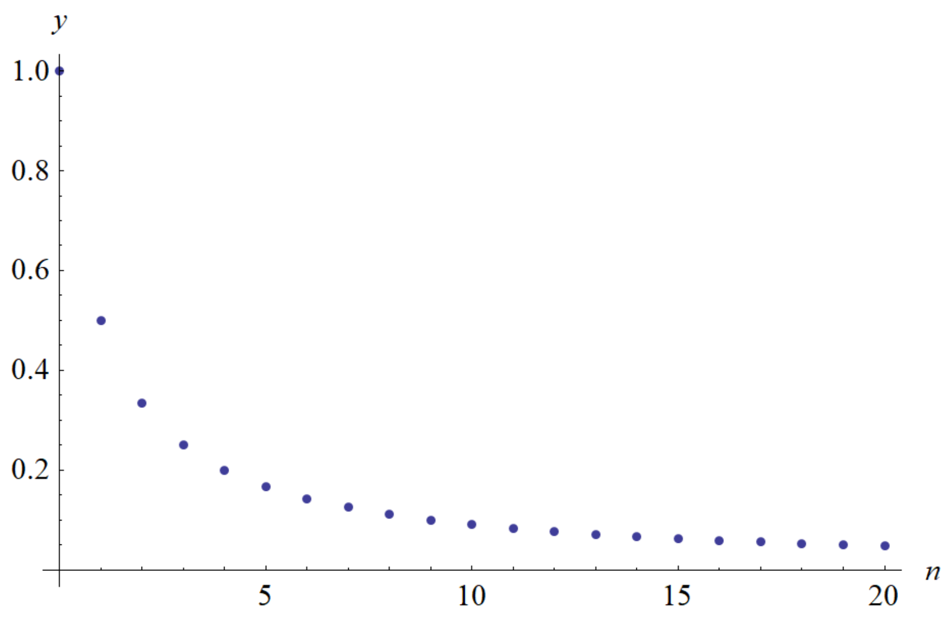
$$y_{n+1} = \frac{n+1}{n+2} y_n$$

Solution

This is a first-order linear difference equation, so it can be solved by iteration.

$$\begin{aligned} n = 0: \quad y_1 &= \left(\frac{1}{2}\right) y_0 \\ n = 1: \quad y_2 &= \left(\frac{2}{3}\right) y_1 = \left(\frac{2}{3}\right)\left(\frac{1}{2}\right) y_0 \\ n = 2: \quad y_3 &= \left(\frac{3}{4}\right) y_2 = \left(\frac{3}{4}\right)\left(\frac{2}{3}\right)\left(\frac{1}{2}\right) y_0 \\ &\vdots \\ y_n &= \frac{n!}{(n+1)!} y_0 = \frac{1}{n+1} y_0 \end{aligned}$$

Below is a plot of y_n versus n for $y_0 = 1$.



Now take the limit of y_n as $n \to \infty$.

$$\begin{aligned} \lim_{n\to\infty} y_n &= \lim_{n\to\infty} \frac{1}{n+1} y_0 \\ &= 0 \end{aligned}$$