Problem 10

In each of Problems 1 through 32, solve the given differential equation. If an initial condition is given, also find the solution that satisfies it.

$$(x^2y + xy - y) + (x^2y - 2x^2)\frac{dy}{dx} = 0$$

Solution

Method Using Separation of Variables

Begin by factoring the left side.

$$(x^2 + x - 1)y + x^2(y - 2)\frac{dy}{dx} = 0$$

Separate variables.

$$x^2(y - 2)\frac{dy}{dx} = -(x^2 + x - 1)y$$

$$\frac{y - 2}{y}\,dy = -\frac{x^2 + x - 1}{x^2}\,dx$$

Integrate both sides.

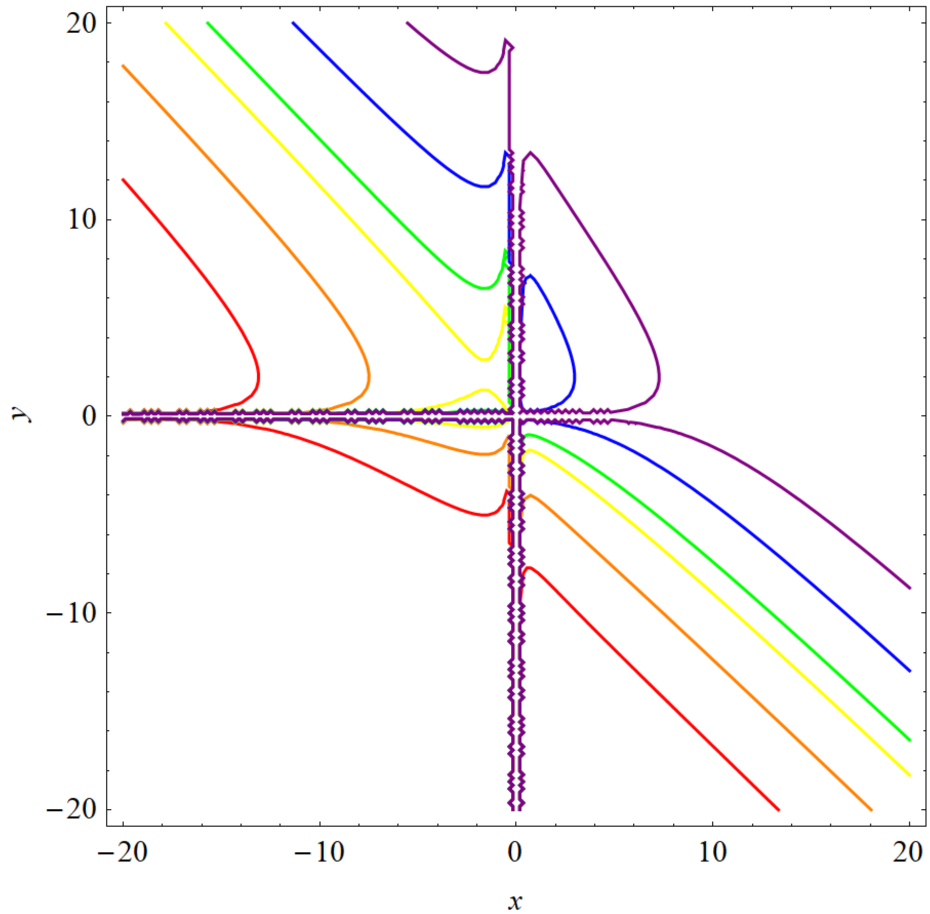
$$\int \left(1 - \frac{2}{y}\right) dy = \int \left(-1 - \frac{1}{x} + \frac{1}{x^2}\right) dx$$

$$y - 2\ln|y| = -x - \ln|x| - \frac{1}{x} + C$$

Absolute value signs were included because the logarithm arguments cannot be negative. Therefore,

$$x + \ln|x| + \frac{1}{x} + y - \ln y^2 = C.$$

Notice also that $y(x) = 0$ satisfies the ODE.



This figure illustrates several solutions of the family. In red, orange, yellow, green, blue, and purple are $C = -10$, $C = -5$, $C = -1$, $C = 1$, $C = 5$, and $C = 10$, respectively.