Problem 13

In each of Problems 1 through 32, solve the given differential equation. If an initial condition is given, also find the solution that satisfies it.

\[
\frac{dy}{dx} = 1 + 2x + y^2 + 2xy^2
\]

**Solution**

Factor the right side.

\[
\frac{dy}{dx} = 1 + 2x + (1 + 2x)y^2 = (1 + 2x)(1 + y^2)
\]

Because the ODE is of the form \( y' = f(x)g(y) \), it can be solved by separating variables.

\[
\frac{dy}{1 + y^2} = (1 + 2x) \, dx
\]

Integrate both sides.

\[
\tan^{-1} y = x + x^2 + C
\]

Therefore,

\[
\tan^{-1} y - x - x^2 = C
\]

This figure illustrates several solutions of the family. In red, orange, yellow, green, blue, and purple are \( C = -10, C = -5, C = -1, C = 1, C = 1.45 \), and \( C = 1.65 \), respectively.

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