Problem 1

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$y'' + 2y' - 3y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} + 2(re^{rt}) - 3(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + 2r - 3 = 0$$

$$(r + 3)(r - 1) = 0$$

$$r = \{-3, 1\}$$

Two solutions to the ODE are $y = e^{-3t}$ and $y = e^t$. Therefore, the general solution is

$$y(t) = C_1 e^{-3t} + C_2 e^t,$$

a linear combination of the two.