Problem 2

In each of Problems 1 through 8, find the general solution of the given differential equation.

$$y'' + 3y' + 2y = 0$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} + 3(re^{rt}) + 2(e^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + 3r + 2 = 0$$

$$(r + 2)(r + 1) = 0$$

$$r = \{-2, -1\}$$

Two solutions to the ODE are $y = e^{-2t}$ and $y = e^{-t}$. Therefore, the general solution is

$$y(t) = C_1 e^{-2t} + C_2 e^{-t},$$

a linear combination of the two.