Problem 7

In each of Problems 1 through 8, find the general solution of the given differential equation.

\[ y'' - 9y' + 9y = 0 \]

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form \( y = e^{rt} \).

\[ y = e^{rt} \rightarrow y' = re^{rt} \rightarrow y'' = r^2e^{rt} \]

Substitute these expressions into the ODE.

\[ r^2e^{rt} - 9(re^{rt}) + 9(e^{rt}) = 0 \]

Divide both sides by \( e^{rt} \).

\[ r^2 - 9r + 9 = 0 \]

\[ r = \frac{9 \pm \sqrt{81 - 4(9)(1)}}{2} = \frac{9 \pm \sqrt{45}}{2} = \frac{9 \pm 3\sqrt{5}}{2} \]

Two solutions to the ODE are

\[ y = \exp \left( \frac{9 - 3\sqrt{5}}{2} t \right) \quad \text{and} \quad y = \exp \left( \frac{9 + 3\sqrt{5}}{2} t \right) . \]

Therefore, the general solution is

\[ y(t) = C_1 \exp \left( \frac{9 - 3\sqrt{5}}{2} t \right) + C_2 \exp \left( \frac{9 + 3\sqrt{5}}{2} t \right) , \]

a linear combination of the two.