Problem 12

In each of Problems 9 through 16, find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as t increases.

$$y'' + 3y' = 0, \qquad y(0) = -2, \quad y'(0) = 3$$

Solution

Since this is a linear homogeneous constant-coefficient ODE, the solution is of the form $y = e^{rt}$.

$$y = e^{rt} \quad \rightarrow \quad y' = re^{rt} \quad \rightarrow \quad y'' = r^2 e^{rt}$$

Substitute these expressions into the ODE.

$$r^2 e^{rt} + 3(re^{rt}) = 0$$

Divide both sides by e^{rt}.

$$r^2 + 3r = 0$$

$$r(r+3) = 0$$

$$r = \{-3, 0\}$$

Two solutions to the ODE are $y = e^{-3t}$ and $y = e^0 = 1$, so the general solution is

$$y(t) = C_1 e^{-3t} + C_2,$$

a linear combination of the two. Differentiate it once with respect to t.

$$y'(t) = -3C_1 e^{-3t}$$

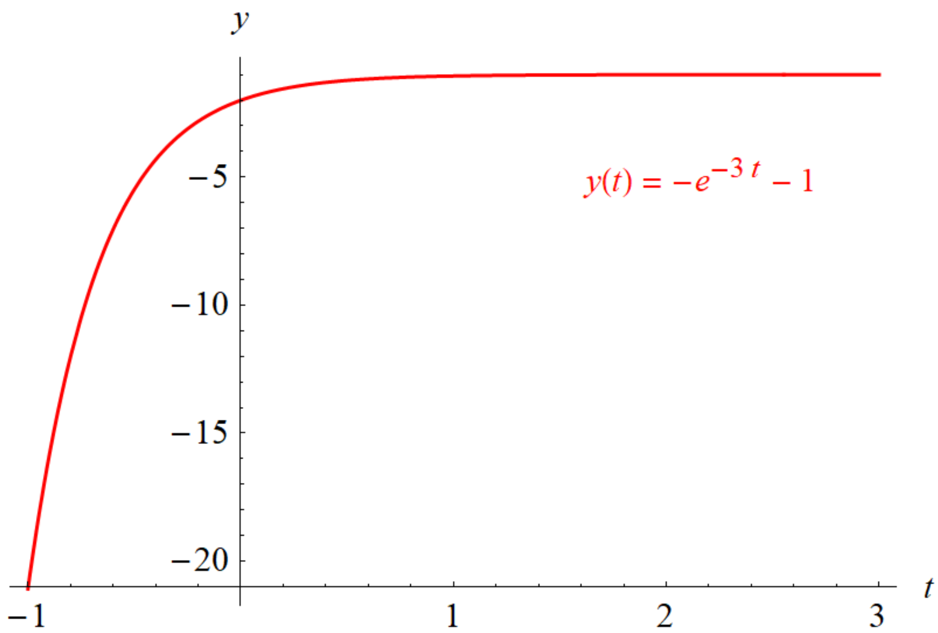
Apply the two initial conditions now to determine C_1 and C_2.

$$y(0) = C_1 + C_2 = -2$$
$$y'(0) = -3C_1 = 3$$

Solving the system of equations yields $C_1 = -1$ and $C_2 = -1$. Therefore,

$$y(t) = -e^{-3t} - 1.$$

This solution converges to -1 as $t \rightarrow \infty$.


y
−5
−10
−15
−20
−1
1
2
3
t
y(t) = −e^{−3t} − 1