

## Problem 17

Find a differential equation whose general solution is  $y = c_1e^{2t} + c_2e^{-3t}$ .

---

### Solution

Work backwards. Two solutions to the ODE are  $y = e^{2t}$  and  $y = e^{-3t}$ . That means that the ODE is linear and homogeneous and has constant coefficients and has solutions of the form  $y = e^{rt}$ . We know that

$$r = \{-3, 2\},$$

so

$$(r + 3)(r - 2) = 0$$

$$r^2 + r - 6 = 0.$$

Multiply both sides by  $e^{rt}$ .

$$r^2e^{rt} + re^{rt} - 6e^{rt} = 0$$

Therefore, a differential equation is

$$y'' + y' - 6y = 0.$$