

## Problem 18

Find a differential equation whose general solution is  $y = c_1e^{-t/2} + c_2e^{-2t}$ .

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### Solution

Work backwards. Two solutions to the ODE are  $y = e^{-t/2}$  and  $y = e^{-2t}$ . That means that the ODE is linear and homogeneous and has constant coefficients and has solutions of the form  $y = e^{rt}$ . We know that

$$r = \left\{ -2, -\frac{1}{2} \right\},$$

so

$$(r + 2) \left( r + \frac{1}{2} \right) = 0$$

$$(r + 2)(2r + 1) = 0$$

$$2r^2 + 5r + 2 = 0.$$

Multiply both sides by  $e^{rt}$ .

$$2r^2e^{rt} + 5re^{rt} + 2e^{rt} = 0$$

Therefore, a differential equation is

$$2y'' + 5y' + 2y = 0.$$