

Problem 6

In each of Problems 1 through 6, find the Wronskian of the given pair of functions.

$$\cos^2 \theta, \quad 1 + \cos 2\theta$$

Solution

The Wronskian of these two functions is

$$\begin{aligned} W &= \begin{vmatrix} \cos^2 \theta & 1 + \cos 2\theta \\ \frac{d}{d\theta}(\cos^2 \theta) & \frac{d}{d\theta}(1 + \cos 2\theta) \end{vmatrix} \\ &= \begin{vmatrix} \cos^2 \theta & 1 + \cos 2\theta \\ 2 \cos \theta(-\sin \theta) & -2 \sin 2\theta \end{vmatrix} \\ &= \cos^2 \theta(-2 \sin 2\theta) - (1 + \cos 2\theta)[2 \cos \theta(-\sin \theta)] \\ &= -2 \cos^2 \theta \sin 2\theta + 2 \sin \theta \cos \theta(1 + \cos 2\theta) \\ &= -2 \cos^2 \theta(2 \sin \theta \cos \theta) + 2 \sin \theta \cos \theta(1 + 2 \cos^2 \theta - 1) \\ &= -4 \cos^2 \theta \sin \theta \cos \theta + 4 \sin \theta \cos \theta \cos^2 \theta \\ &= 0. \end{aligned}$$